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Quadratic Voting Can Be Inefficient

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Abstract

I consider a collective choice problem with asymmetric information about policy effects to explore the limits of the efficiency of Quadratic Voting (QV), a social choice mechanism in which voters buy votes for or against a proposal at a quadratic cost and the outcome with the most votes wins. In some cases, it is natural to assume that individuals are asymmetrically informed about the effects of legislation and therefore their valuations of legislation. For instance, anti-corruption legislation is the most beneficial to taxpayers and the most detrimental to corrupt officials when the government is corrupt, but government officials have better information than taxpayers about how corrupt the government is. I provide an example of a setting in a large population where QV does not achieve approximate utilitarian efficiency despite majority voting achieving full efficiency. The efficiency of the QV equilibrium I analyze is less than 50%. In this example, a society considers an anti-corruption policy that protects taxpayers from corruption and hurts the incomes of corrupt officials. Officials know whether they are corrupt, but taxpayers are uncertain about whether corruption is occurring. When there is a minority with superior information and preferences opposed to the majority and this minority knows the stakes are high, it can expend resources to take the election.

JEL No. D47, D61, D71, C72, P16

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1 Introduction

QV is a decision-making mechanism where voters purchase votes for or against each option at a cost that is equal to the square of the number of votes purchased for that option. QV holds promise as a practical, approximately efficient system for making social decisions. Fea- tures of the mechanism that make it practical include budget-balance, the minimal amount of information it requires the designer to have, and the simplicity of the reports it requires from agents (Eguia et al. 2019). Budget-balance means that the mechanism can be run without any external funding. The simplicity of the mechanism means that real people facing time con- straints are able to participate. Approximate efficiency is the best feasible efficiency property for a budget-balanced mechanism since full efficiency is not possible for a budget-balanced mechanism (see Proposition 5.1 and Corollary 5.1 in Börgers 2015). Furthermore, QV is reasonably resilient to collusion and fraud (Weyl 2017). Theoretical results prove QV is approximately efficient in a variety of settings. Theoretical work establishes that QV performs well with binary decisions and private values. Lalley and Weyl (2019) show QV is approximately efficient in large populations when voters have independent private values. Chandar and Weyl (2019) approximate QV equilibria in finite-population examples and find QV performs reasonably well. Weyl (2017) finds that QV is robust to a number of deviations to the independent private values Bayes-Nash model such as aggre- gate uncertainty. To the best of my knowledge, there is no private-value theoretical example where the welfare loss of QV is more than 10%. Additional theoretical results establish efficiency in other settings. Eguia et al. (2019) show that QV approaches efficiency as the population size gets large in a full-information setting with multiple alternatives. Posner and Weyl (2017) prove the best equilibrium of QV dominates the best equilibrium of plurality voting in any pure common-interest setting, including ones involving private information. I now introduce the terminology of of Holmström and Myerson (1983) to discuss the point at which efficiency is evaluated.

Efficiency can be evaluated before agents have learned private information (*ex ante* efficiency), after each agent has learned his own private information but before the private information of other agents has been revealed (*ex interim* efficiency), or after each agent has learned the private information of all other agents (*ex post* efficiency). In the theoretical analyses discussed previously, efficiency is calculated from an *ex ante* perspective.

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Empirical tests of QV confirm it is relatively efficient in private-value settings. Firstly, a lab experiment confirms QV is close to efficient in an independent private value setting. Goeree and Zhang (2017) study the performance of QV in a lab experiment in an independent private value setting and find the welfare loss under QV is no more than 15.2% under any of their treatments and QV outperforms plurality voting. This welfare loss is calculated from an *ex ante* perspective. In the experiments of Goeree and Zhang (2017), QV is *ex ante* efficient while plurality voting was not. Groups of participants were allowed to choose between the two mechanisms for the final rounds and 73% of the groups chose QV.

Furthermore, tests of QV in the political context suggest that QV can help people select bundles of policies. Casella and Sanchez (2019) run an experiment and find that QV has the potential to improve political decision-making. A limitation of their conclusions is that they only evaluate welfare by asking voters how important a policy is to them and using importance as a proxy for magnitude of valuations. Choices under majority voting are used to determine the direction of valuations. Although this framework is appropriate for evaluating efficiency in private- value contexts, it is inappropriate for evaluating efficiency in common-value contexts. One of the policies in this study plausible has a large common-value component. That policy is to increase required pre-tenure experience for teachers from two to five years. Although almost all voters desire high-quality public education. many voters could be uncertain whether this proposal would increase the quality of public education. This measure could plausibly improve the quality of public education by giving principals more time to evaluate teachers. It is also plausible that this policy could degrade the quality of public education by making harder to recruit quality teachers. In common-value context, this method of evaluating efficiency does not always yield accurate results when comparing mechanisms. For instance, consider the following example. The electorate consists of a bad teacher, an uniformed voter, and an education economist. The bad teacher loses $10,000 if the teacher tenure proposal passes. From the uniformed voter's perspective, there is uncertainty over whether teacher tenure increases the quality of public education. Teacher tenure could either increase or decrease the quality of public education. The uninformed voter is the father of 5 children and therefore places a high importance on the quality of public schools. If teacher tenure increases the quality of public education, teacher tenure delivers benefits of $10,000 to the education economist and a benefit of $25,000 to the uninformed voter. If it decreases the quality of the education system, the decrease in the quality of public education costs the education economist $10,000 and the uninformed voter $25,000. The education economist knows whether teacher tenure increases or decreases the quality of the education system but is unable to communicate with the uniformed voter. The uninformed voter believes that there is a 60% chance teacher tenure will decrease the quality of public education and a 40% chance that teacher tenure will increase the quality of public education. Formally, chance moves at the beginning of the game and chooses the state where teacher tenure decreases the quality of public education with 60% probability. The education economist observes the state but the uninformed voter does not. The structure of the model is common knowledge. Under majority voting, the bad teacher votes against teacher tenure and the education economist votes for the policy that maximizes the quality of public education. The uninformed voter's optimal strategy is to defer the decision to the better-informed voter with identical preferences (Feddersen and Pesendorfer 1996). He can accomplish this by voting for teacher tenure to counteract the vote of the bad teacher. Notably, the uninformed voter votes for a policy he expects to be harmful. Such a vote never hurts him because in equilibrium, the education economist always casts the deciding vote against teacher tenure whenever teacher tenure is harmful. Consider using the efficiency measurement technique of Casella and Sanchez (2019). Assume that voters report an importance that is proportional to their willingness to pay for their ex post favorite policy instead of their ex post least favorite policy. Then, if teacher tenure decreases the quality of public education, the decision under majority rule is efficient even though the efficiency measurement technique of Casella and Sanchez (2019) calculates that it is inefficient. Despite this limitation, this experiment suggests that QV could improve political decision-making. Politicians also had a positive experience with QV when using it to aggregate their preferences. The Democratic caucus of the Colorado State House of Representatives used QV to decide which bills were most important for them to fund and reported that QV gave them a better signal than the mechanism they used prior to adopting QV (Rogers 2019). It is important to determine how generally QV is efficient. In this work, I explore an example with asymmetric information about policy effects that shows QV can be quite inefficient.

In the proceeding analysis, I analyze the *ex ante* inefficiency of QV and the *ex ante* efficiency of plurality voting. In some cases, the policy decision is not important. The reason QV is much worse than plurality voting in the next example is that plurality voting always makes the right decision when the policy is important while QV rarely does so.

2 Baseline Model

I use a model that is a special case of the model of Eguia et al. (2019). A society considers an anti-corruption proposal, which is supported by taxpayers and opposed by corrupt government officials. There are a finite number of individuals N in the society: $N_t > 0$ taxpayers and $N_g > 0$ government officials. Let $k = \frac{N_t}{N_g}$ denote the ratio of taxpayers to government officials. There are two states: one in which the government is not corrupt and one in which the government is corrupt. The state is known to government officials but only the prior distribution of the state is known to taxpayers. Let p be the prior probability that the government is corrupt. The anti-corruption proposal prevents corruption from occurring if it passes. If the government is corrupt and the anti-corruption proposal fails, government officials steal money from taxpayers. Each corrupt government official steals a total of s. A loss of $\theta s N_g$ is divided evenly among all taxpayers, where $\theta > 1$. θ includes the money stolen, the efficiency cost of raising public funds with distortionary taxes, harm inflicted by cuts to cost-effective public services, and the cost of wasteful corrupt activities. No anti-corruption proposal is the status quo. Voter i chooses an action a_i. Positive values of a_i can be interpreted as the number of votes purchased for the proposal. Negative values of a_i correspond to buying $|a_i|$ votes against the proposal. A monetary payment of a_i^2 is required for an action of a_i: $|a_i|$ votes have a cost of a_i^2. Let A be an indicator for the passage of the alternative. The probability that the alternative passes is $\frac{e^{\phi \sum_{i=1}^{N} a_i}}{1+e^{\phi \sum_{i=1}^{N} a_i}}$, where $\phi > 0$.

Importantly, this probability is increasing in $\sum_{i=1}^{N} a_i$. This probabilistic passage of the proposal is primarily a tractable way to approximate a deterministic decision rule that passes the alternative when the alternative gets a positive number of net votes and fails the alternative when the alternative gets a negative number of net votes. When φ becomes large, this passing probability closely approximates the deterministic rule. This functional form for the probability of an alternative's victory is also used in Eguia et al. (2019). Lalley and Weyl (2019) also assume that the payoff is a continuous function of the vote total. In addition, this model is an accurate description of reality if the system for counting votes sometimes makes errors and the probability that the wrong winner is declared is higher when the election is close. There is some evidence that errors sometimes occur in real elections. For instance, Keating (2002) finds that Al Gore would have won the 2000 presidential election if votes had been counted accurately, but George Bush won according to the initial count and a court order prevented a complete recount. Furthermore, the election was close: there were 105,372,669 votes that were officially valid and Gore needed only 538 more votes in Florida to win. The revenue raised from a voter is redistributed evenly among all other voters. Normalize the utility of neither losing or gaining money to 0. Let S be an indicator for whether the government is corrupt. The utility of taxpayer i is

$$-\left(1-\frac{e^{\phi \sum_{j=1}^{N} a_j}}{1+e^{\phi \sum_{j=1}^{N} a_j}}\right)\frac{S\theta s}{k} - a_i^2 + \frac{1}{N-1}\sum_{j\neq i} a_j^2.$$

The utility of government official i is

$$\left(1-\frac{e^{\phi \sum_{j=1}^{N} a_j}}{1+e^{\phi \sum_{j=1}^{N} a_j}}\right) Ss - a_i^2 + \frac{1}{N-1}\sum_{j\neq i} a_j^2.$$

2

2.1 An example of inefficiency in large populations

It is optimal for government officials to bid zero when the government is not corrupt because in that case, they do not care about which alternative is selected. Corrupt government official i's first-order condition is

$$0 = -s\frac{\left(1+e^{\phi \sum_{j=1}^{N} a_j}\right)\phi e^{\phi \sum_{j=1}^{N} a_j} - e^{\phi \sum_{j=1}^{N} a_j}\phi e^{\phi \sum_{j=1}^{N} a_j}}{\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2} - 2a_i = -s\phi\frac{e^{\phi \sum_{j=1}^{N} a_j}}{\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2} - 2a_i.$$

Therefore,

$$2a_i = -s\phi\frac{e^{\phi \sum_{j=1}^{N} a_j}}{\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2}.$$

Hence,

$$a_i = -s\phi \frac{e^{\phi \sum_{j=1}^{N} a_j}}{2\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2}.$$

Let $a_g(1)$ denote the equilibrium action of a corrupt government official and let a_t denote the equilibrium action of a taxpayer. Then,

$$a_g(1) = -s\phi \frac{e^{\phi(N_g a_g(1) + N_t a_t)}}{2\left(1 + e^{\phi(N_g a_g(1) + N_t a_t)}\right)^2}.$$

Taxpayer i's first-order condition is

$$0 = \frac{p\theta s}{k} \frac{\left(1 + e^{\phi \sum_{j=1}^{N} a_j}\right)\phi e^{\phi \sum_{j=1}^{N} a_j} - e^{\phi \sum_{j=1}^{N} a_j}\phi e^{\phi \sum_{j=1}^{N} a_j}}{\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2} - 2a_i = \frac{p\theta s\phi}{k} \frac{e^{\phi \sum_{j=1}^{N} a_j}}{\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2} - 2a_i.$$

Therefore,

$$\frac{p\theta s\phi}{k} \frac{e^{\phi \sum_{j=1}^{N} a_j}}{\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2} = 2a_i.$$

Hence,

$$a_i = \frac{p\theta s\phi}{k} \frac{e^{\phi \sum_{j=1}^{N} a_j}}{2\left(e^{\phi \sum_{j=1}^{N} a_j}\right)^2}.$$

Therefore,

$$a_t = \frac{p\theta s\phi}{k} \frac{e^{\phi(N_g a_g(1) + N_t a_t)}}{2\left(1 + e^{\phi(N_g a_g(1) + N_t a_t)}\right)^2}.$$

Therefore, if the government is corrupt, the vote total for the proposal is

$$N_g\left(-s\phi \frac{e^{\phi(N_g a_g(1) + N_t a_t)}}{2\left(1 + e^{\phi(N_g a_g(1) + N_t a_t)}\right)^2}\right) + N_t\left(\frac{p\theta s\phi}{k} \frac{e^{\phi(N_g a_g(1) + N_t a_t)}}{2\left(1 + e^{\phi(N_g a_g(1) + N_t a_t)}\right)^2}\right)$$

$$= \left(-N_g + \frac{p\theta}{k} k N_g\right) s\theta \frac{e^{\phi(N_g a_g(1) + N_t a_t)}}{2\left(1 + e^{\phi(N_g a_g(1) + N_t a_t)}\right)^2}$$

$$= (p\theta - 1) N_g s\theta\phi \frac{e^{\phi(N_g a_g(1) + N_t a_t)}}{2\left(1 + e^{\phi(N_g a_g(1) + N_t a_t)}\right)^2} < 0.$$

If $p\theta < 1$, then $p\theta - 1 < 0$, so the proposal is likely to fail when the government is corrupt even though the unique efficient social choice when the government is corrupt is for the proposal to succeed. The anti-corruption proposal is likely to pass only when the government is not corrupt. *Expected inefficiency*, defined following Lalley and Weyl (2019) to be the unique negative linear function of expected utility that lies in the unit interval, is

$$EI = \frac{e^{\phi(p\theta-1)N_g s\theta\phi \frac{e^{\phi(N_g a_g(1)+N_t a_t)}}{2\left(1+e^{\phi(N_g a_g(1)+N_t a_t)}\right)^2}}}{1+e^{\phi(p\theta-1)N_g s\theta\phi \frac{e^{\phi(N_g a_g(1)+N_t a_t)}}{2\left(1+e^{\phi(N_g a_g(1)+N_t a_t)}\right)^2}}} = \frac{e^{\phi^2(p\theta-1)N_g s\theta \frac{e^{\phi(N_g a_g(1)+N_t a_t)}}{2\left(1+e^{\phi(N_g a_g(1)+N_t a_t)}\right)^2}}}{1+e^{\phi^2(p\theta-1)N_g s\theta \frac{e^{\phi(N_g a_g(1)+N_t a_t)}}{2\left(1+e^{\phi(N_g a_g(1)+N_t a_t)}\right)^2}}}.$$

This definition of efficiency defines a 0% efficient decision to be the decision that gives the lowest utilitarian social welfare and a 100% efficient decision that gives the highest possible social welfare. A mechanism is X% efficient if using it instead of making the worst possible decision realizes X% of the surplus that could have been gained from switching from the worst possible decision to the best possible decision. In this context, expected inefficiency is the probability that the proposal fails when the government is corrupt. If $k > 1$, then plurality voting implements the socially optimal outcome.

So, when $p\theta - 1 < 0$ and $k > 1$, QV achieves less than 50% efficiency but majority voting achieves 100% efficiency. Note that the low efficiency of QV persists in this case even when the population is large. One can also verify from this example that when there is uncertainty about the effects of a policy, it is possible in some states data about voters' expected values for a policy are not informative about the efficiency of such a policy because the better-informed voters have more accurate expectations. If the government is corrupt and $p < 1$, the total valuation of the officials for the policy is $-sN_g$ while the total expected valuation of taxpayers for the policy is $p\theta sN_g$ so the sum of all voters' expected valuations for the policy is negative despite the policy being efficient in that case.

2.2 When does QV outperform majority voting?

For some parameter values, QV will outperform majority voting. QV will select the efficient outcome most of the time if $p > 1$. Majority voting will be inefficient if $N_g > N_t$. Therefore, QV significantly outperforms majority voting when $p > 1$ and $N_g > N_t$. This model reinforces the point made in other QV papers such as Weyl (2017) that QV will outperform majority voting when the majority of voters support a policy that voters are fairly certain is inefficient.

3 A costly anti-corruption policy

Next, I add a cost to the anti-corruption policy. This cost represents expenditures on activities such as auditing and monitoring to ensure corruption does not occur. The *ex ante* expected welfare loss due to corruption is $(\theta - 1)\, sN_g$. Let c be the cost of the anti-corruption policy. I assume that c is evenly distributed among all individuals. Let $a_g(0)$ denote the action of a clean government official. The utility of taxpayer i is $p\left(1-\frac{e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}\right)\frac{-\theta s}{k}-\left(\frac{e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}+(1-p)\frac{e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}\right)\frac{c}{N}-a_i^2+\frac{1}{N-1}\sum_{j\neq i}a_j^2$.

The utility of corrupt government official i is

$$\left(1-\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}{1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}\right)s-\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}{1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}\frac{c}{N}-a_i^2+\frac{1}{N-1}\sum_{j\neq i}a_j^2.$$

The utility of clean government official i is

$$-\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}}{1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}}\frac{c}{N}-a_i^2+\frac{1}{N-1}\sum_{j\neq i}a_j^2.$$

I use first-order conditions to derive necessary conditions for an equilibrium. Corrupt government official i's first-order condition is

$$0 = -\frac{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\right)\phi e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}-e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\phi e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\right)^2}$$

$$\left(s+\frac{c}{N}\right)-2a_i=-\frac{\phi e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\right)^2}\left(s+\frac{c}{N}\right)-2a_i.$$

Therefore,

$$-\frac{\phi e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\right)^2}\left(s+\frac{c}{N}\right)=2a_i.$$

Hence,

$$a_i = -\frac{\phi e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}{2\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\right)^2}\left(s+\frac{c}{N}\right).$$

Thus, an equilibriun condition is

$$a_g(1) = -\frac{\phi e^{\phi\left(N_t a_t+N_g a_g(1)\right)}}{2\left(1+e^{\phi\left(N_t a_t+N_g a_g(1)\right)}\right)^2}\left(s+\frac{c}{N}\right).$$

Define $x = e^{\phi\left(a_t+(N_t-1)a_t+N_g a_g(1)\right)}$. Corrupt government official i's second-order condition is

$$0 > -\left(s+\frac{c}{N}\right)\phi\frac{(1+x)^2\phi x - x2(1+x)\phi x}{(1+x)^4} - 2 = -\left(s+\frac{c}{N}\right)\phi^2\frac{(1+x)x-2x^3}{(1+x)^3} - 2$$
$$= -\left(s+\frac{c}{N}\right)\phi^2\frac{x-x^2}{(1+x)^3} - 2$$
$$= -\left(s+\frac{c}{N}\right)\phi^2\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)} - e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)^2}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\right)^3} - 2$$
$$= -\left(s+\frac{c}{N}\right)\phi^2\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)} - e^{2\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(1)\right)}\right)^3} - 2.$$

Thus, an equilibrium condition is

$$0 > -\left(s+\frac{c}{N}\right)\phi^2\frac{e^{\phi\left(N_t a_t+N_g a_g(1)\right)} - e^{2\phi\left(N_t a_t+N_g a_g(1)\right)}}{\left(1+e^{\phi\left(N_t a_t+N_g a_g(1)\right)}\right)^3} - 2.$$

Taxpayer i's first-order condition is

$$0 = \frac{p\theta s}{k}\frac{\phi e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}\right)^2} - \frac{pc}{N}\frac{\phi e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}\right)^2}$$
$$-(1-p)\frac{c}{N}\frac{\phi e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}\right)^2} - 2a_i.$$

Therefore,

$$p\left(\frac{\theta s}{k}-\frac{c}{N}\right)\frac{\phi e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}\right)^2} - (1-p)\frac{c}{N}\frac{\phi e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}\right)^2} = 2a_i.$$

Thus,

$$a_i = p\left(\frac{\theta s}{k}-\frac{c}{N}\right)\frac{\phi e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{2\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}\right)^2} - (1-p)\frac{c}{N}\frac{\phi e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}}{2\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}\right)^2}.$$

Therefore, an equilibrium condition is

$$a_t = p\left(\frac{\theta s}{k}-\frac{c}{N}\right)\frac{\phi e^{\phi\left(N a_t+N_g a_g(1)\right)}}{2\left(1+e^{\phi\left(N_t a_t+N_g a_g(1)\right)}\right)^2} - (1-p)\frac{c}{N}\frac{\phi e^{\phi\left(N_t a_t+N_g a_g(0)\right)}}{2\left(1+e^{\phi\left(N_t a_t+N_g a_g(0)\right)}\right)^2}.$$

Taxpayer i's second-order condition is

$$0 > p\left(\frac{\theta s}{k} - \frac{c}{N}\right)\phi^2 \frac{e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)} - e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)^2}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}\right)^3}$$
$$- (1-p)\phi^2 \frac{c}{N}\frac{e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)} - \left(e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}\right)^2}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}\right)^3} - 2$$
$$= p\left(\frac{\theta s}{k} - \frac{c}{N}\right)\phi^2 \frac{e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)} - e^{2\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(1)\right)}\right)^3}$$
$$- (1-p)\phi^2 \frac{c}{N}\frac{e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)} - e^{2\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}}{\left(1+e^{\phi\left(a_i+(N_t-1)a_t+N_g a_g(0)\right)}\right)^3} - 2.$$

Therefore, an equilibrium condition is

$$0 > p\left(\frac{\theta s}{k} - \frac{c}{N}\right)\phi^2 \frac{e^{\phi\left(N_t a_t+N_g a_g(1)\right)} - e^{2\phi\left(N_t a_t+N_g a_g(1)\right)}}{\left(1+e^{\phi\left(N_t a_t+N_g a_g(1)\right)}\right)^3}$$
$$- (1-p)\phi^2 \frac{c}{N}\frac{e^{\phi\left(N_t a_t+N_g a_g(0)\right)} - e^{2\phi\left(N_t a_t+N_g a_g(0)\right)}}{\left(1+e^{\phi\left(N_t a_t+N_g a_g(0)\right)}\right)^3} - 2.$$

The first-order condition of clean government official *i* is

$$0 = -\frac{c}{N}\frac{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\right)\phi e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)} - e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\phi e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\right)^2}$$
$$- 2a_i = -\frac{c}{N}\phi\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\right)^2} - 2a_i.$$

Therefore,

$$a_i = -\frac{c}{N}\phi\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}}{2\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\right)^2}.$$

Therefore, an equilibrium condition is

$$a_g(0) = -\frac{c}{N}\phi\frac{e^{\phi\left(N_t a_t+N_g a_g(0)\right)}}{2\left(1+e^{\phi\left(N_t a_t+N_g a_g(0)\right)}\right)^2}.$$

The second-order condition of clean government official *i* is

$$0 > -\frac{c\phi^2}{N}\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)} - \left(e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\right)^2}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\right)^2} - 2$$
$$= -\frac{c\phi^2}{N}\frac{e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)} - e^{2\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}}{\left(1+e^{\phi\left(a_i+N_t a_t+(N_g-1)a_g(0)\right)}\right)^2} - 2.$$

Therefore, an equilibrium condition is

$$0 > -\frac{c\phi^2}{N}\frac{e^{\phi\left(N_t a_t + N_g a_g(0)\right)} - e^{2\phi\left(N_t a_t + N_g a_g(0)\right)}}{\left(1 + e^{\phi\left(N_t a_t + N_g a_g(0)\right)}\right)^2} - 2.$$

The first-order conditions are not sufficient because they only require the derivative of a voter's strategy with respect to his action to be zero. It is possible for local minima to exist that satisfy the first-order conditions but not the second-order conditions. For example, suppose s = 468, c = 500, $\phi = 1$, N_g = 2, N_t = 2498, p = .5, and $\theta = {}^{1213}/_{468}$. $a_g(1) = -30.1$, $a_g(0) = -1.3 \times 10^{-28}$, and $a_t = 0.025$ satisfy the necessary conditions but are not an equilibrium. The corrupt government official's second-order condition is not satisfied and spending nothing on voting is a profitable deviation for a corrupt government official. I find equilibria by solving the first-order conditions and verifying the second-order conditions. Corrupt government officials vote against the anti-corruption proposal because they wish to steal money and avoid paying the cost of enforcing the anti-corruption proposal. Clean government officials also vote against the anti-corruption proposal because they wish to avoid paying the cost of the proposal. Taxpayers may vote for the proposal because they hope to pass it when the government is corrupt but may also vote against it because they do not want to pay the proposal's cost. I consider a numerical example based on the anti-corruption experiment of Olken (2007). This experiment increased the probability of an audit from 0.04 to 1 and measured the change in the amount of money stolen from road projects. I consider the proposal that would increase the probability that a government official was audited. Olken (2007) measures the amount of materials used in a road and compares it to the amount of materials used in an road that was built honestly. A corrupt official might steal money that was supposed to be spent on materials and then reduce the quality of the road by building it with fewer materials. Around 25% of officials used more materials than Olken (2007) estimates are required. I assume that due to measurement error and road-specific heterogeneity, another 25% of officials needed fewer materials than Olken (2007) estimates were needed to build the road properly. Therefore, I assume that there is a 50% chance officials building a road are clean and a 50% chance they are corrupt. Hence, I set p = .5. Olken (2007) estimates that audits cost $500, reduced average theft by $468, and increased the benefits of the average road by $1,213. Based on Olken (2007), I set N = 2,500, N_g = 7, and c = 500, and θ = 2.58. I set ϕ = 1. Olken (2005) finds that audits reduce theft by 8% of the value of the road on average but only 6.5% of the value of the road when the village head does not face an upcoming election within 2 years. I analyze the case where the village head does not face an upcoming election within the next two years. To model the case where the village head does not face an upcoming election within the next two years, I set s = 109. Taxpayers vote for the proposal and government officials vote against it, causing it to pass with 14.2% probability when the government is corrupt and 99.999999999999999997% probability when the government is clean. Efficiency is only 8.3%. The equilibrium efficiency is worse than both the efficiency of never auditing (41.5%) and the efficiency of always auditing (58.5%). A policy of never auditing and zero transfers would make everyone better off relative to the equilibrium outcome.

Intuitively, what happens from any taxpayer's perspective is that other taxpayers are going to buy more than enough votes to pass the proposal when the government is clean and enough votes to have a chance of passing the proposal when the government is corrupt. In this case, it is too expensive to buy enough votes against the proposal to block it if the government is clean. Voting for the proposal is very unlikely to make a difference if the government is clean but has a much more significant effect on the chance of passing the proposal if the government is corrupt, so the taxpayer has an incentive to buy votes for the proposal. Taxpayers face an expected loss of 1129 in this equilibrium. They are better off if the government is never audited and no money is transferred during the political process, since they face an expected loss of only 984 in that case. They are also better off if the government is always audited and no money is transferred during the political process, since they face a loss of only 499 in that case. Notably, QV achieves a lower overall welfare than dictatorship by any individual in this example.

An extension to the model in the spirit of Patty and Penn (2017) would require proposed policies to be written by some member of society. I model the proposal process as a simultaneous game. Each member of society decides whether or not to propose this policy. The policy is considered if it has been proposed by at least one individual. The number of government officials proposing the policy is observable. Taxpayers have no incentive to propose this policy since they do poorly under QV when the proposal is introduced. There is an equilibrium where no government officials submit proposals. In this equilibrium, taxpayers believe that a proposal from the government official means the government is clean.

In that case, the taxpayers do not make QV payments that are larger than those made by government officials, so government officials do not gain any benefit from making the proposal. In practice, it is plausible that nobody in the village has an incentive to make this proposal. The auditor might have an interest in ensuring plenty of audits are done, but in this context the auditor is not a resident of the village.

Furthermore, the benefits to individual workers of reducing the theft of wages through audits are also small. Hence, a lack of proposal incentives mitigates effect of the poor performance of QV on this proposal. Taxpayers vote against an anti-corruption proposal if it is sufficiently expensive relative to its benefits. I now present some alternative assumptions that make the audits less cost-effective to create an example where everyone votes against the audits. I use the Newbery and Stern (1987) estimate of 2.15 for the marginal cost of public funds rather than the value of 1.4 from Olken (2007), raising the cost of the anti-corruption proposal rises to $c = 751$. Olken (2007) uses a 5% social discount rate. I raise the social discount rate to 7%, a value that Lopez (2008) argues is justifiable. Raising the social discount rate lowers θ to 2.17. In that case, everyone votes against the proposal.

3.1 The Bayesian Underdog Effect in the corruption context

? Finds that when private values are affiliated rather than independent, some inefficiency results because supporters of unlikely winners believe a tie is more likely than supporters of likely winners. He terms this phenomenon a Bayesian Underdog Effect. Because individuals know their own values, the efficiency loss is small. The efficiency loss can happen only rarely because it is caused by one alternative being more likely to win. Similarly, corrupt officials believe the chance their votes are important is relatively high, taxpayers believe the chance their votes are important is moderate, and clean officials believe the change their votes are pivotal is relatively low. These differing beliefs increase the vote expenditures of corrupt officials and decrease the vote expenditures of clean officials. As a result, QV over-weights the preferences of corrupt officials and under-weights the preferences of clean officials. Therefore, corrupt officials are usually not audited and clean officials are almost always audited.

3.2 Incentives to reveal expertise under QV

This example shows that QV does not provide experts who are not trusted by the rest of the population to reveal expertise when they are indeed trustworthy. Government officials are experts at determining whether the government is corrupt. Furthermore, the clean government officials and taxpayers have a common interest: neither group wants to waste money auditing a clean government. The result that QV may perform worse than majority voting in a partially-shared-interest setting is surprising given that prior work suggests that QV will perform well in such settings. For example, Posner and Weyl (2017) suggest that QV will outperform majority voting in a setting with partially shared interests and asymmetric information.

Elections for abusable positions

QV may pose a problem for the elections of officials to positions where corrupt officials can profit. Corrupt officials would steal from the public for personal gain if elected and therefore have a strong monetary incentive to get themselves elected. Under QV, they may attempt to win the election either by buying votes for themselves or colluding with other corrupt individuals that they are working with. To get a sense for the potential magnitude of this problem, consider a real-world example of corruption. From 2002 to 2008, Kwame Kilpatrick stole $4.5 million ($5.3 million in 2019 dollars) from Detroit (Cohn 2016). Credit constraints might not be an issue for corrupt individuals trying to buy elections because banks sometimes participate in corruption (Gradel and Simposon 2015). The number of corruption convictions among Detroit mayors suggests corruption is rare: only 2 of the 77 mayors of Detroit have been convicted of corruption. An adverse selection problem may occur because the mayoral candidates who are willing to pay the most to become mayor are likely the ones who intend to steal money. Intuitively, it seems impossible for a society to choose honest candidates over corrupt candidates when only the candidates themselves know whether they are corrupt. When a mechanism designer lacks information, he is subject to the constraint that he must induce agents to provide truthful information. Such mechanisms are the best an uninformed mechanism designer can do (Myerson 1979). Consider the following model: A social planner has must choose a candidate for office and has two candidates available. Each of the candidates is corrupt with probability p determined independently. Each candidate's type is private information. An honest benefit gets a benefit of w for holding office, representing the benefits legally available to the office-holder. A corrupt candidate gets a benefit of $w + s$, where s represents the benefits a corrupt candidate can illegally obtain from office.

The social planner seeks to minimize the probability a corrupt candidate is chosen. By the Revelation Principle (Myerson 1979), it suffices to consider direct mechanisms. Each candidate type maximizes the probability it attains office, so to induce truthful reporting, the mechanism designer must give clean and corrupt candidates the same probability of attaining office. When multiple people engage in corruption and the social planner knows which people are aware whether the government is corrupt, the planner can create a direct mechanism that has an efficient equilibrium. The planner can punish anyone reporting the government is clean if one informed person has re- ported the government is corrupt.

However, this mechanism has inefficient equilibria as well, including one where all corrupt individuals report the government is clean and thus prevent the planner from discovering corruption and one where informed individuals randomize. When the informed individuals randomize, the planner fails to gain accurate information about corruption and is sometimes even forced to punish some individuals. Moreover, it could be difficult in practice for a planner to know which individuals are informed.

Conclusion

The above example shows that QV can be inefficient when a poorly informed, unsuspecting majority can sometimes be overwhelmed by an opposition minority that catches it off guard. This example is similar to the adverse selection problem in markets for private goods studied by Akerlof (1970). In both models, efficiency is low because the poorly informed agents benefit most from changing the allocation when the well-informed agents lose the most from changing the allocation. It is like the right to an honest government being sold by the government officials. This right is worthless when the government is already honest. When the government is corrupt, this right is costly for the officials to give up but even more valuable for the taxpayers. When the informational advantage of the corrupt officials is high relative to the gains from trade, taxpayers cannot pass the legislation with high probability when it is valuable.

Because QV is only approximately efficient in private-value and pure-common-interest settings, those are the only settings where a strong theoretical justification for using QV exists. It is plausible, however, that QV would be approximately efficient whenever voters had common rather than conflicting interests. If information that causes one voter to like a policy more also causes other voters to like that policy more, the problem in the above example does not arise. Although QV might be better than majority voting in cases involving asymmetric information about policy effects, because there are cases where majority voting is efficient and QV is not approximately efficient, it is an empirical matter whether QV or majority voting is better in applications.

The structure of this example suggests that we should be especially wary about using QV in a setting where a minority with superior information has interests that are opposed to that of society as a whole. Gun rights and corruption control are two such issues: in both, potential criminals have the best information on whether the proposed law would deter antisocial activity. A tax reform aimed at closing tax loopholes to extract more tax revenue from a minority of wealthy people is another example where QV might do poorly. The wealthy are likely better informed about how a tax reform targeting them will affect them than the general population and their interest in keeping their own wealth conflicts with the majority's interest in raising public revenue. The wealthy would put up more opposition to reforms that are surprisingly effective at collecting taxes from them.

The variant of QV used also determines the risk of an inefficient minority victory due to an information asymmetry. QV with money carries the highest risk of an inefficient minority victory because a minority's influence is limited only by that minority's willingness and ability to spend money. The variant of QV recommended for short-and medium-term use by Posner and Weyl (2018) using voice credits that are evenly distributed among individuals is less prone to inefficient minority victories because a minority's influence is limited by the amount of voice credits its members have received. Determining whether QV is efficient in practice in real-world settings that involve both conflicting interests and uncertainty about policy effects is an important area for future research. Another interesting area for future research is the effectiveness of information-providing institutions for reducing the amount of inefficiency in QV caused by poor information. It is possible that efficiency losses from poor information can be mitigated if credible information is readily accessible to voters.

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