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The Empirical Study of National Defense Expenditure and Economic Growth: A Game Theory Approach

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1. INTRODUCTION

National defense expenditure refers to the grand total of resources that a country commits to the construction of national defense over a specific period of time. The fundamental issue that many studies on national defense expenditure tend to focus on is identifying the proportion of a country's total social economic resources (i.e., GDP) that is allotted to national defense construction and the ideal configuration of resource allocation. This has been the prevalent area for two reasons: 1) national defense expenditure has always been one of the government's largest expenditure items, and 2) it is useful to understand the correlation between national defense expenditure and economic growth in order to facilitate a balanced development of both the economy and national defense construction. However, the construction of an ideal growth scheme for national defense expenditure that facilitates the development of a country's economy and the construction of national defense has remained one of the untapped areas. So far, scholars fall into two major schools of thoughts on this subject. The first approach analyzes the correlation between economic growth and national defense expenditure by using historical data on economic growth and national defense expenditure to determine the quantitative correlation between the two and to identify the extent of the impact of national defense expenditure on economic growth (Barro, 1990). Another approach is the combination of longitudinal and transverse cross sections. This works by performing a transverse comparison on the historical data of a country's national defense expenditure with relevant historical data from other nations to deduce the growth model for national defense expenditure that exists.

While both approaches have their merits, neither is flawless. For example, many research conducted according to the former approach tends to focus on the externality of national defense expenditures. This is because the quantification of national defense while avoiding the analysis of the security benefits that national defense expenditure could bring is very difficult. As for the latter approach, it operates on the premise that the conclusions of relevant studies must be backed with a specific level of prevalence. Indeed, while both approaches are valuable, an approach has yet to be proposed that accurately reflects the development and demands for national defense security for a specific country at a specific period of time.

In this study, we examine data from Taiwan, South Korea, Japan, India, and Israel to attempt to fulfill the following objectives: 1) to construct a game-theoretic model to reveal the interactive relationship between the national defense expenditure and economy in both Taiwan and South Korea. The model should shed light on solutions that would allow decision making unit (DMUs) to minimize the impact of national defense expenditure on their economy while accommodating the demands for national defense and security within reasonable limits of resource allocation, 2) to further analyze the relative utilization efficiency for the national defense expenditure of seven DMUs to ensure that an efficient analysis can be made. We attempt to answer the following three research questions: 1) what is the ideal way to achieve balance between national defense expenditure and the cost of this to economic construction? 2) what is the more effective way of measuring the true value of national defense expenditure or the equilibrium solution? 3) can a model based on game theory and data envelopment analysis (DEA) be used together to provide a starting point for constructing a theoretical framework and an in-depth discussion of how to undertake an objective evaluation of the equilibrium between national defense construction, economic construction, and resource configuration?

We begin with the brief introduction, followed by the literature review. Data analysis and the discussion of results are then provided in the end.

2. THEORETICAL BACKGROUND

2.1 Defense expenditure, military threat, and economic growth

It is commonly believed that national defense expenditure is a non-productive cost that detracts from other valuable economic activities. Empirical studies have, however, demonstrated that there is a strong correlation between national defense expenditure and economic growth. Benoit (1978) discovered that national defense expenditure had positive effects on economic growth. His findings were replicated by many researchers such as Macnair et al. (1995), Brumm (1997), and Murdoch et al. (1997). Other studies have identified a negative correlation between national defense expenditure and economic growth (Mintz & Hung, 1990; Ward & Davis, 1992; Lipow & Antinori; 1995). It is argued that higher levels of spending on defense resulted in slower economic growth or even decline. Surprisingly, literature has revealed no significant correlation between national defense budget and economic growth (Mintz & Stevenson, 1995; Landau, 1996; DeRouen, 1995).

Research has found the relationship between defense expenditure and economic growth is highly complex. Deger and Sen (1995) revealed that national defense expenditure affected economic growth through supply and demand. Zou (1995) found that military threats from hostile nations had no long-term effect on domestic economic growth, while Chang et al. (1996) discovered that there was a correlation between an anticipated military threat from hostile nations and the development of domestic national defense.

Shieh et al. (2002a) empirically examined the impact from the factors of foreign military threat, government finance, and national defense expenditure, on a country's economic growth. They found that relative risk aversion is a critical variable that determines how the domestic economy's long-term equilibrium and short-term dynamic is affected by a foreign military threat. Shieh et al. (2002b) argued that national defense expenditure affects economic growth through three channels: the spin-off effect, crowding-out effect, and resource mobilization effect. By examining these three channels, they found that there was a positive correlation between threats from hostile nations and the long- and short-term health of a nation's economy. Aizenman and Glick (2003) posited that, as the military threat from hostile nations grows stronger, a country's national defense expenditure and long-term economic growth also develops at a rate that is directly proportional.

2.2 Defense expenditures and military capability

The military budget of a hostile nation has been found by some researchers to affect the budget assigned to defense in the country that is being approached in a hostile way. In their 2001 study, Chang et al. (2001) defined military defense capability as a country's capacity for defense and deterrence, while Klein (2004) contended that military capability is manifested as a nation's military strength and its capability to defend its sovereignty. Kollias et al. (2004) posited that the scale of a national defense budget depends on the hostility from other nations. Lambelet et al. (1979) and Brams (1985) believed that the degree of military capabilities ultimately affects the demands for national defense budget. Meanwhile, Burgess (1988) claimed that national defense budget planning is correlated to a military threat, while Cline (1980) conducted an empirical study to analyze the relationship between the defense budget of the offensive nation and that of the defensive nation.

3. METHODOLOGY AND DATA ACQUISITION

In this study, we use the game-theoretic model developed by Lai et al. (2005) to derive an appropriate formula for assessing national defense expenditure. The subsequent formula is then used to generate the national defense expenditure equilibrium solution for Taiwan and South Korea. In the end of this section, data envelopment analysis model (DEA model) is developed to examine the relative efficiency between national defense budget (input) and military capability (output).

3.2 Game theory

For the purpose of this study, we assume that the Nash equilibrium is the state of equilibrium between national economy and national security in a complete information static game. It functions under the premise that relevant governmental departments and the national defense department strive to prioritize economic growth, prioritize national security, or achieve balanced development between national defense and economy.

As indicated in table 1, there are two equilibrium solutions to the game. In this case, the equilibrium solution of “balance between national defense and economy” is the logical choice. During the process of the game, it became clear that the government would always opt to drive up national defense expenditure with minimum adverse impact on economic growth, despite the fact that both the government and national defense department tended to pursue the state’s general strategic objectives. The military tended to opt for a solution that would achieve the highest satisfaction of the national defense security requirements. These tendencies shaped the state of balance for the game’s equilibrium solution.

TABLE 1 Nash Equilibrium of National Defense Security and National Economy

National defense security

		Prioritize national defense over economy	Prioritize economy over national defense	Balance between national defense and economy
National economy	Prioritize national defense over economy	-1,1	-1,-1	-1,0
	Prioritize economy over national defense	1,1	1,-1	1,0
	Balance between national defense and economy	0,1	0,-1	0,0*

3.3 Malmquist index

In recent years, the Malmquist index has become a popular approach to productivity measurement within the non-parametric literature. Caves et al. (1982) propose a Malmquist productivity index that is relative to a single technology ϕ^t (in (1)) or ϕ^{t+1} (in (2)), considering *n* decision making units (DMUs) in time period *t* that use inputs $X^t \in R_+^m$ to produce output $Y^t \in R_+^s$, as follows:

$$M_o^t = \frac{D_0^{t+1}\left(X^{t+1},Y^{t+1}\right)}{D_o^t\left(X^t,Y^t\right)}$$

$$M_o^{t+1} = \frac{D_0^{t+1}\left(X^{t+1},Y^{t+1}\right)}{D_o^t\left(X^t,Y^t\right)}$$

$D_o^t\left(X^t,Y^t\right)$ is the output distance function, which is determined on the basis of technology ϕ^t as the reciprocal to the maximal feasible expansion of Y^t producible from input X^t. The values of M_o^t and M_o^{t+1} may be greater, equal, or smaller than one, depending on whether productivity growth, stagnation, or decline has occurred between periods *t* and *t* + 1. In general, M_o^t and M_o^{t+1} yield different productivity numbers since their reference technologies are different.

Färe et al. (1994) successfully applied the Malmquist index his study. The findings contributed significantly to the development of this index by relaxing the efficiency assumption and using DEA models (Charnes et al., 1978) for the calculation of the distance functions embodied in it. Note that an output distance function coincides with the DEA measure of technical efficiency. This development makes it far easier to implement linear programming models to compute the Malmquist index. Färe et al. (1994) defined the output-oriented productivity index as the geometric mean of the two Malmquist indexes referring to the technology at time periods *t* and *t+1*, (1) and (2) respectively, yielding the following Malmquist measure of productivity:

$$I^{t+1,t} = \left[\frac{D_0^t\left(X^{t+1},Y^{t+1}\right)}{D_0^t\left(X^t,Y^t\right)} \times \frac{D_0^{t+1}\left(X^{t+1},Y^{t+1}\right)}{D_0^{t+1}\left(X^t,Y^t\right)}\right]^{1/2}$$

$$I^{t+1,t} = \frac{D_0^{t+1}\left(X^{t+1},Y^{t+1}\right)}{D_o^t} \times \left[\frac{D_0^t\left(X^{t+1},Y^{t+1}\right)}{D_0^{t+1}\left(X^{t+1},Y^{t+1}\right)} \times \frac{D_0^t\left(X^t,Y^t\right)}{D_0^{t+1}\left(X^t,Y^t\right)}\right]^{1/2}$$

The ratio outside the bracket measures the technical efficiency change between time periods *t* and *t* + 1. The geometric mean of the two ratios inside the bracket captures the technological change (or shift in technology) between the two periods, evaluated by the input-output levels at t (X^t, Y^t) and at *t* + 1 (X^{t+1}, Y^{t+1}). Overall, improvements in productivity yield Malmquist indexes ($I^{t+1,t}$) with values that are greater than unity; conversely, declines in productivity yield Malmquist indexes ($I^{t+1,t}$) with values that are smaller than unity.

3.4 Data acquisition

The data used in the DEA estimation comprised of 336 observations from seven DMUs over 48 years. The DEA model can evaluate the relative efficiency scores of these DMUs by linear programming based on selected variables. Therefore, the interpretation and efficiency scores are affected by the selection of inputs and outputs. To ensure the validity of the research both data availability (an empirical criterion) and a literature survey are used in the study (See Table 2, Figure 1).

TABLE 2 Decision Factors for DMUs

	TWN	KOR	TWN-E	KOR-E	JAPAN	INDIA	ISRAEL
Population & National income	23,119,772 USD31,900	49,044,797 USD 19,231	23,119,772 USD 31,900	49,044,797 USD19,231	127,767,944 USD33,800	1,147,995,898 USD3,737	7,370,000 USD26,200
Military threat	From China	From North Korea	From China	From North Korea		From Pakistan, China and Bangladesh	Lebanon, Syria, Jordan and Egypt
Economy	Export oriented	Export oriented	Export oriented	Export oriented	Highly developed economy	The primary output nation for information industry	The most developed nation in the Middle East in terms of industrialization and economy
Geographical location	An island located to the east of Asia and northwest Pacific Ocean	Southern part of the Korean Peninsula	An island located to the east of Asia and northwest Pacific Ocean	Southern part of the Korean Peninsula	An island nation located to the west of Pacific Ocean and east of Eurasia	The largest nation in Southern Asia	Surrounded by Lebanon (north), Syria, Jordan (east) and Egypt (southwest)
Colonial history	Under Japanese rule from 1896-1945	Under Japanese rule from 1905-1945	Under Japanese rule from 1896-1945	Under Japanese rule from 1905-1945	Defeated during WWII; under Allied occupation until 1952.	Under British rule as a colony from 1849-1950	The Jewish people were constantly under exile and rule by ancient empires such as Persia. Became a sovereign nation on May 14, 1948.

Figure 1. Geographical location of DMUs

3.5 Data and Input-output variables descriptions

In this study, the model developed by Lai et al. (2005) is adopted to derive the formula for national defense expenditure in order to formulate the national defense expenditure equilibrium solution for Taiwan and South Korea between 1961 and 2008. Next, the Malmquist index from the DEA model is used to examine the relative efficiency between national defense budget (input) and military capability (output). The relative efficiencies of Taiwan, South Korea, Japan, India, and Israel between 1961 and 2008 are then compared. Definitions of relevant variables and research framework are shown in Table 3 and Figure 2.

TABLE TABLE 3 Definition of Variables

Type	Variable	Indicator	Definition	Source
Game model	Taiwan and South Korea’s actual GDP	GDP	GDP is the basic measure of a country’s overall economic output. It is an important indicator of a country (region)’s economic status.	OECD Database
Game model	Taiwan and South Korea’s labor force	LABOUR	The population of people above 15 years of age and ready for employment, including the employed and unemployed.	OECD Database
Game model	Taiwan and South Korea’s actual capital deposit	CAPITAL	Capital deposit or deposit capital. From the perspective of corporate capital operation, it refers to all existing capital resources in a corporation’s possession.	OECD Database

Game model	Taiwan and South Korea's national defense technology R&D budget	DEFENSE BUDGET FOR R&D	Annual national defense budget that goes to technology R&D	OECD Database
Game model	Taiwan and South Korea's national defense expenditure	Military Budgets	True value of national defense expenditure from each fiscal year	Military Balance
DEA Input	Military Budgets	Military Budgets	The true value of each nation's annual national defense expenditure	Military Balance
			Equilibrium solution to Taiwan and South Korea's national defense expenditure game	
DEA Output	Military Capability	Defensive military strength	No. of active troops / Area of territory	Military Balance
		Organizational structure ratio	No. of troops in the navy and air force / No. of active troops	

Source: Compiled by the author

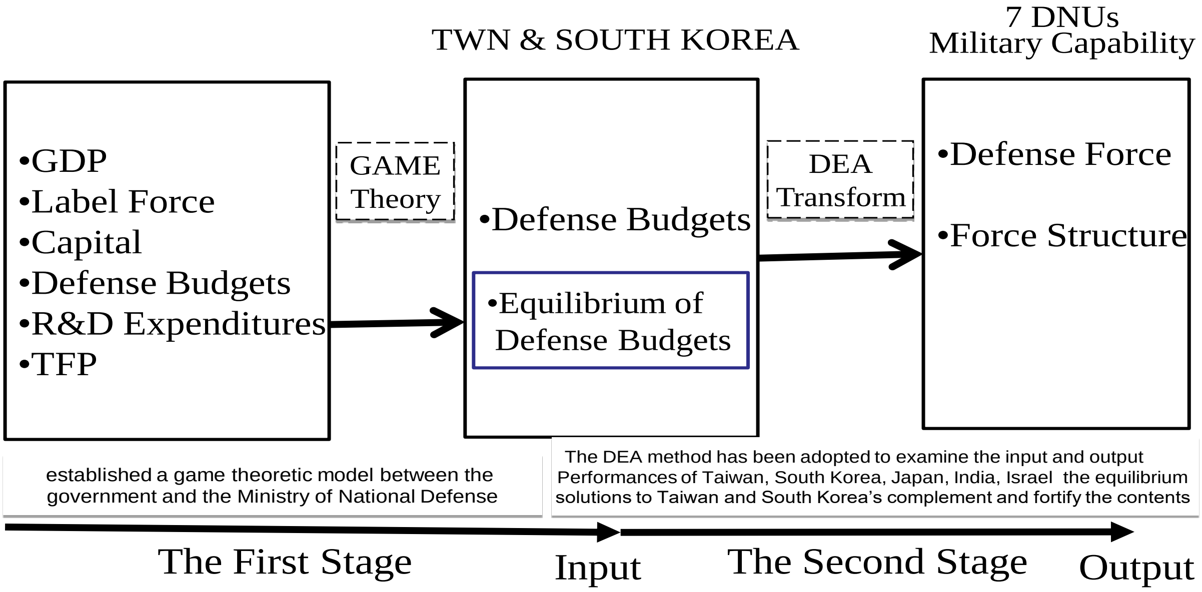

Figure 2 The Research Framework

4. MODEL DERIVATION

4.1Analysis of the national defense capital model

From an economic perspective, the effective military deterrence capability (S_t) of an army comes from two primary components: the capital deposit of its national defense (M_t) (including weaponry, equipment, and human resources), and military management and operational knowledge (KM_t) (including military tactic theories and organizational systems). This can be represented as the function: St = F (M_t, KM_t). Let r_t be the percentage of national defense technology research and investment in national defense expenditure and R_t be the proportion of national defense budget demand in the GDP. Since national defense technology research and investment is usually excluded from the calculation of national defense capital increment, ergo the formula for national defense capital may be represented as follows:

$$M_t = M_{t-1} + Y_t R_t (1-r_t) - \lambda \sum Y_{t-i} R_{t-i} (1-r_{t-i}) \quad (i=1,2,\cdots),$$

where λ represents the rate of depreciation. With this established, we can represent the national defense security production function as follows:

$$S_t = F\left[M_t = M_{t-1} + Y_t R_t (1-r_t) - \lambda \sum Y_{t-i} R_{t-i} (1-r_{t-i}), KM_t\right] \quad (1)$$

4.2 National defense expenditure and economic growth

From the theoretical perspective of production function, national defense expenditure will boost TFP. Using the Cobb-Douglas Production Function $Y_t = AK^{\alpha} L_t^{\beta}$ as the basis, we can derive the following formula for the economic growth rate:

$$\Delta Y/Y_t = \Delta A/A_t + \alpha \Delta K/K_t + \beta \Delta L/L_t \quad (2)$$

Let $\Delta Y/Y_t = y_t, \Delta A/A_t = \mathrm{a}_t, \Delta K/K_t = k_t, \Delta L/L_t = l_t$

Then, $y_t = \mathrm{a}_t + \alpha k_t + \beta l_t$

Let the annual national defense budget be $D_t = Y_t R_t$; then, the comprehensive effect of the national defense expenditure on economic growth can be represented as follows:

$$y(D_t) = \mathrm{a}(D_t) + \alpha k(D_t) \quad (3)$$

(3) The actual format for equation (3) can also be represented as follows:

$$y(R_t, r_t) = \mathrm{a}(r_t) + \alpha k(R_t, r_t)$$

Where a *(r_t)* is obtained from the production function $Y_t = AK^{\alpha} L_t^{\beta}$ and the TFP for technical advancement refers to the productivity of a set of traditional input, it can be computed using the following formula:

$$TFP = Y/(K^{\alpha} L^{\beta}) \quad (4)$$

By using the growth rate formula to modify equation (2), we get the following:

$$g_{TFP} = y - ak - \beta l \quad (5)$$

In his study on endogenous growth theory, Gerace (2002) perceived endogenous technical advancement as the primary proponent of economic growth; with this in mind, we also propose the following:

$$g_{TFP} = \Omega(RD/Y) \quad (6)$$

Equation (6) is derived under the premise where knowledge capital depreciation is overlooked, and the assumption that the net change (dA) in the deposit of knowledge capital is equivalent to the sum of investment for research and development (R&D), where $\Omega \equiv (dY/dA)$ represents knowledge capital's marginal productivity. Likewise, let r_t represent the proportion of the national defense expenditure taken up by R&D; if we assume Ω_1 to be the marginal productivity for national defense R&D investment, then $RD = Y_t R_t r_t$ and a *(r_t)* may be represented as follows:

$$\mathrm{a}(r_t) = \Omega_1 (Y_t R_t r_t)/Y_t = \Omega_1 R_t r_t.$$

National defense expenditure capital deposit K_t is derived from prior capital deposit K_{t-1} and the net investment function I_t. In this paper, the following formula has been chosen for the derivation of capital deposit:

$$K_t = K_{t-1} + I_t - \lambda \sum I_{t-i} \qquad (i = 1,2,\cdots), \quad (7)$$

where λ stands for the rate of capital depreciation. From equation (7), the decrement in capital assets in year t caused by national defense expenditure can be represented as follows:

$$\Delta K_t = I_t + (1-\lambda)\sum I_{t-i} = \mathrm{a} Y_t R_t (1-r_t) + \mathrm{a}(1-\lambda) \sum_{i=1,2,\cdots} Y_{t-i} R_{t-i} (1-r_{t-1})$$

thus, $$\alpha k(R_t, r_t) = \left[\mathrm{a} Y_t R_t (1-r_t) + \mathrm{a}(1-\lambda) \sum_{i=1,2,\cdots} Y_{t-i} R_{t-i} (1-r_{t-1}) / k_t \right].$$

In economics, the assessment of factors' contribution to economic growth is usually represented as a contribution rate. In this case, the contribution rate of national defense expenditure to economic growth can be represented as $e_D = y(R_t, r_t)/y_t$, and therefore,

$$e_D = y(R_t, r_t)/y_t = \Omega R_t(1-r_t)/y_t - \left[aY_t R_t(1-r_t) + a(1-\lambda) \sum_{i=1,2,\cdots} Y_{t-i} R_{t-i}(1-r_{t-1})/k_t y_t \right].$$

4.3 Model derivation

Let us assume that a government's goal is to satisfy fundamental national defense security demands so that national defense expenditure can provide the greatest comprehensive contribution to economic development. Let us also assume that as the principle party responsible for national defense expenditure and the provider of national defense security, a national defense department is primarily interested in maximizing national defense security. Let π be the ratio of national defense expenditure against overall financial spending for LDCs, π_0 be the ratio of minimum expenditure for people's livelihoods, R_0 be the ratio of minimum national defense expenditure, and r_0 be the basic military expenditure and maintenance spending.

The increase in a nation's defense budget has been found by a number of studies to be intricately related to the demands for military equipment development. This is especially true for nations that are actively pursuing weaponry upgrade (e.g., Taiwan); the national defense budget increase for such nations will depend on the equipment that is purchased and relevant R&D. Thus, the government has a binding power over the national defense budget R_t and the relevant R&D r_t proposed by the national defense department.

Let the binding function be

$$R_t = \mu r_t + R_0 \qquad (8)$$

Equation (8) represents the basic national defense budget that a national defense department will receive when it does not take part in weaponry/equipment development (including R&D investment). In order to derive the maximum national defense security output, the optimized strategy function may be represented as

$$Max\{S_t = F[M_t = M_{t-1} + Y_t R_t(1-r_t) - \lambda \sum Y_{t-i} R_{t-i}(1-r_{t-i}), KM_t]\}$$

$$s.t. \quad R_t = \mu r_t + R_0$$

$$R_0 < R_t < \pi - \pi_0$$

$$0 < r_t < 1 - r_0. \qquad (9)$$

On the other hand, in order for a government to facilitate economic growth, the optimized strategy function may be represented as

$$Max\left\{ \Omega R_t(1-r_t)/y_t - \left[aY_t R_t(1-r_t) + a(1-\lambda) \sum_{i=1,2,\cdots} Y_{t-i} R_{t-i}(1-r_{t-1})/k_t y_t \right] \right\} \qquad (10)$$

$$s.t. \quad R_t = \mu r_t + R_0$$

$$R_0 < R_t < \pi - \pi_0$$

$$0 < r_t < 1 - r_0.$$

By satisfying the government's demands—as represented in equation (10)—we can derive the function of equilibrium for both the government and national defense department. We assume the first order derivative condition to be 0 for r_t in equation (9).

$$\partial S/\partial r_t = (\partial F/\partial M_t)(\partial M_t/\partial r_t) + (\partial F/\partial M_t)(\partial KM_t/\partial r_t) = (\partial F/\partial M_t) Y_t(\mu + R_0 + 2\mu r_t) = 0$$

We then get $r_t = (\mu - R_0)/2\mu$;

by plugging $r_t = (\mu - R_0)/2\mu$ into $R_t = \mu r_t + R_0$,

we arrive at $R_t = (\mu + R_0)/2$.

By plugging in $b_t = \Omega_1/y_t$; $c_t = \alpha Y_t$; $d_t = \alpha\{(1-\lambda)\sum Y_{t-i} R_{t-i}(1-r_{t-i})\}/K_t y_t \quad (i = 1,2,\cdots)$,
we get

$$\varphi(R_t, r_t) = (b_t + c_t) R_t r_t - c_t R_t - d_t. \qquad (11)$$

By converting equation (11) into a function for μ, we get the following function:

$$\varphi(\mu) = 0.25(b_t + c_t)(\mu - R^2/\mu) - 0.5c_t\mu - 0.5c_t R_0 - d_t \qquad (12)$$

If we let the first-order condition for the previous equation be 0, we get the following function:

$$\partial\varphi(\mu)/\partial\mu = 0.25(b_t + c_t)(\mu - R^2/\mu) - 0.5c_t = 0 .$$

We can then obtain the solution $\mu = [(c_t + b_t)/(c_t - b_t)]^{1/2} R_0$.

With an incentive scheme, the equilibrium solution for national defense budget and economic construction would be

$$r_t^* = 1/2 - 1/[4(c_t + b_t / c_t - b_t)]^{1/2} \qquad (13)$$

$$R_t^* = \{[(c_t + b_t)/(c_t - b_t)]^{1/2} + 1\} R_0 / 2 .$$

If we ignore the range of change during the span of two consecutive years, we can represent the growth rate for national defense budget as follows:

$$\upsilon_t = Y_t R_t^* / Y_{t-1} R_{t-1}^* - 1 .$$

By substituting the equilibrium solution $r_t^* = 1/2 - 1/[4(c_t + b_t / c_t - b_t)]^{1/2}$ and $R_t^* = \{[(c_t + b_t)/(c_t - b_t)]^{1/2} + 1\} R_0 / 2$ back into the original equation, we will be able to derive the growth rate for national defense budget as follows:

$$\upsilon_t = (1 + y_t)\{[(c_t + b_t)/(c_t - b_t)]^{1/2} + 1\} / \{[(c_{t-1} + b_{t-1})/(c_{t-1} - b_{t-1})]^{1/2} + 1\} - 1 . \qquad (14)$$

5. EMPIRICAL ANALYSIS OF THE MODEL

5.1 Marginal productivity Ω_1 of national defense R&D investment

Now that we have established the model $Y_t = AK^{\alpha} L_t^{\beta}$, we may begin to apply it to calculate Taiwan's and South Korea's capital output elasticity α and labor output elasticity β before computing the results for $TFP = Y/(K^{\alpha} L^{\beta})$.

By adopting the model $LnY = b + aLnK + \beta LnL + \mu$ for the regression analysis, we can derive both α and β for Taiwan, as follows:

$$LnY = \underset{2.398}{2.420} + \underset{5.601}{0.439} LnK + \underset{1.917}{0.346} LnL + 0.350 \underset{1.136}{D}$$

$R^2 = 0.899$ *Significance-F*=148.840.

When this model is applied to South Korea, the capital output elasticity α and labor output elasticity β are found to be as follows:

$$LnY = \underset{-14.27}{-18.1} + \underset{1.687}{0.211} LnK + \underset{14.901}{4.726} LnL + 0.268 \underset{1.623}{D}$$

$R^2 = 0.979$ *Significance-F*=650.670

TABLE 4 TWN Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			Correlations			Collinearity Statistics	
		B	Std. Error	Beta	T	Sig.	Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	2.420	1.009		2.398	.020					
	LCAPITAL	.439	.078	.555	5.601	.000	.926	.621	.251	.205	4.882
	LLABLE	.346	.180	.284	1.917	.061	.914	.262	.086	.092	10.868
	D	.350	.266	.148	1.316	.194	.852	.183	.059	.160	6.245

TABLE 4 TWN Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	2.420	1.009		2.398	.020					
	LCAPITAL	.439	.078	.555	5.601	.000	.926	.621	.251	.205	4.882
	LLABLE	.346	.180	.284	1.917	.061	.914	.262	.086	.092	10.868
	D	.350	.266	.148	1.316	.194	.852	.183	.059	.160	6.245

a. Dependent Variable: LGDP

TABLE 5 TWN Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.948[a]	.899	.893	.39054	.899	148.840	3	50	.000

a. Predictors: (Constant), D, LCAPITAL, LLABLE

TABLE 6 KOREA Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	-18.100	1.268		-14.270	.000					
	LNK	.211	.125	.063	1.687	.099	.790	.252	.038	.356	2.807
	LNL	4.726	.317	.877	14.901	.000	.988	.917	.334	.145	6.909
	D	.268	.165	.073	1.623	.112	.855	.243	.036	.251	3.983

a. Dependent Variable: LNY

TABLE 7 KOREA Model Summary

Mode	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.989[a]	.979	.977	.27847	.979	650.670	3	42	.000

a. Predictors: (Constant), D, LNK, LNL

From these equations, it is apparent that the model generates relatively accurate estimates for match and coefficient significance. We can therefore establish that during the period between 1956 and 2009, Taiwan's capital output elasticity α = 0.439 and its labor output elasticity β = 0.346, whereas during the period between 1961 and 2008, South Korea's α = 0.138 and β = 5.127. In contrast, Japan's capital output elasticity α = 0.317 and labor output elasticity β = 10.650 between 1961 and 2009. If we substitute α and β from Taiwan and Korea into the formula $TFP = Y/(K^{\alpha}L^{\beta})$, we are then able to calculate the three countries' TFP growth rate from 1956 (or 1961) to 2009, as shown in Table 8:

TABLE 8 Growth rate of Taiwan and South Korea's TFP over the years

YEAR	Taiwan TFP	Korea TFP	YEAR	Taiwan TFP	Korea TFP	YEAR	Taiwan TFP	Korea TFP
1957	0.009896	N/A	1974	-0.008400	0.001876	1991	0.005441	0.012674
1958	0.013687	N/A	1975	-0.006950	0.001989	1992	-0.001860	0.013552
1959	0.043149	N/A	1976	0.017222	0.002576	1993	0.003498	0.023820
1960	0.309356	N/A	1977	-0.199310	0.003119	1994	0.001768	0.027191
1961	-0.299710	N/A	1978	0.138126	0.004244	1995	0.006231	0.028615
1962	0.006221	0.000455	1979	0.002583	0.005232	1996	0.007995	0.029561
1963	-0.000530	0.000721	1980	-0.000170	0.005501	1997	0.000349	0.029903
1964	0.015206	0.000787	1981	0.006737	0.006450	1998	7.57E-05	0.030825
1965	-0.000290	0.000779	1982	0.005539	0.006732	1999	0.004597	0.029822
1966	-0.001640	0.000727	1983	0.012291	0.007061	2000	0.003263	0.031987
1967	-0.001260	0.000778	1984	0.013269	0.007394	2001	0.011468	0.032786
1968	-0.001480	0.000814	1985	0.010656	0.008055	2002	0.004547	0.041107
1969	0.002606	0.000987	1986	0.007527	0.008787	2003	0.005446	0.043515
1970	0.008217	0.001022	1987	0.006985	0.007861	2004	-0.004080	0.048496
1971	0.003073	0.001213	1988	0.000839	0.009297	2005	0.000846	0.055175
1972	0.008428	0.001231	1989	0.003201	0.010994	2006	0.008457	0.033637
1973	0.011758	0.001466	1990	0.001867	0.011099	2007	0.006893	0.036223

5.2 Estimation of national defense research and development budget weighting ($R_t r_t$) in the GDP

5.3 Data correction and estimation

Given that Taiwan's period of national defense technology R&D reform did not take place until 1980, when we calculate the marginal productivity of Taiwan's national defense R&D investment, we will perform the regression analysis on data collected prior to 1980. Let M_t be Taiwan's annual national defense R&D budget, and $M_t = R_t r_t + (R_2/Y)_t$. We can modify equation (9) as follows:

$$g_{TFP} = a + b_i M_{t-i} + cD_t + \mu \qquad (16)$$

In this equation, D_t represents the impact of major state policies on technological advancement, and it exists in the model as a dummy variable (prior to 1980, $D_t = 0$ and after 1980, $D_t = 1$) and *u* represents. Let the actual national defense R&D budget be RM_t; then, plug 0.28735 as the value for the national defense R&D budget back into the regression formula; this gives the following results:

$$g_{TFP} = a + bR_t r_t + c\,D + \mu$$

$$g_{TFP} = \underset{16.250}{0.029} - \underset{-4.788}{1.083} R_t r_t + \underset{1.215}{0.005} D$$

R^2 = 0.560 and the t-value for the variable $R_t r_t$ = -4.788, which indicates significance. Therefore, we have derived the value of marginal productivity for Taiwan's national defense R&D to be -1.083. The t-value for political factor *D* is at 1.215, which indicates a positive impact on technological advancement, although the impact is relatively weak.

TABLE 9 TWN Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.748[a]	.560	.516	.003483396	.560	12.709	2	20	.000

Predictors: (Constant), D, Rtrt

TABLE 10 TWN Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	.029	.002		16.250	.000					
	Rtrt	-1.083	.226	-.840	-4.788	.000	-.726	-.731	-.710	.716	1.397
	D	.005	.004	.213	1.215	.239	-.234	.262	.180	.716	1.397

a. Dependent Variable: TFP

In 1982, owing to the petroleum energy crisis, South Korea's former president Chun Doo-Hwan saw the need to develop the country's hi-tech sectors. Therefore, 1982 is chosen as the dividing point for South Korea.

$$g_{TFP} = \mathrm{a} + b_i M_{t-i} + cD_t + \mu \tag{17}$$

$$g_{TFP} = \underset{8.612}{0.00001677} - \underset{-1.228}{0.00062}\, R_t r_t + \underset{2.837}{0.000003089}\, D$$

R^2 = 0.280 and the t-value for the significance test for the variable $R_t r_t$ are found to be -1.228. Although this is insignificant, it is still safe to assume that the variable has a specific level of influence. From this finding, we have derived the value of marginal productivity for South Korea's national defense R&D to be -0.00062. The t-value for political factor *D* in Korea's case is at 2.837, which suggests a relatively weak positive impact on technological advancement.

TABLE 11 KOERA Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	1.677E-8	.000		8.612	.000					
	Rtrt	-6.200E-7	.000	-.180	-1.228	.226	-.385	-.182	-.157	.759	1.318
	D	3.089E-9	.000	.417	2.837	.007	.505	.393	.363	.759	1.318

a. Dependent Variable: TFP

TABLE 12 KOERA Model Summary

Model	R	R	Adjusted	Std. Error of the	Change Statistics

		Square	R Square	Estimate	R Square Change	F Change	df1	df2	Sig. F Change
1	.529[a]	.280	.247	.000000003214433	.280	8.557	2	44	.001

By substituting original data into equation $y_t = (Y_t / Y_{t-1}) - 1$, we can calculate the true economic growth rate and growth rate of the national defense budget for Taiwan and South Korea under equilibrium.

TABLE 13 TWN and KOERA's economic growth, defense budget growth and national defense budget in equilibrium

YEAR	TWN economic growth	TWN defense budget growth	TWN defense budget in equilibrium growth	YEAR	KOREA economic growth	KOREA defense budget growth	KOR defense budget in equilibrium growth
1957	0.072545917	0.290863891	N/A	1957	N/A	N/A	N/A
1958	0.082989412	**0.272026962 >**	0.152587780	1958	N/A	N/A	N/A
1959	0.250030176	0.021953066 <	0.401285100	1959	N/A	N/A	N/A
1960	0.062033576	0.085555556 <	0.448308010	1960	N/A	N/A	N/A
1961	0.063161450	**0.141589901 >**	-0.189424230	1961	N/A	N/A	N/A
1962	0.079037003	0.038254632 <	0.095756520	1962	0.990185676	0.1818181820	N/A
1963	0.093539337	0.078583765 <	0.113319234	**1963**	0.180601093	**0.407692308 >**	0.3054606894
1964	0.121989402	0.083800374 <	0.148879618	1964	0.031437655	-0.185792350 <	0.1405177040
1965	0.111350000	**0.148731839 >**	0.130432695	1965	0.041095890	-0.020134228 <	0.1511965774
1966	0.089132160	**0.163879957 >**	0.103818020	1966	0.282051282	0.232876712 <	0.4176342859
1967	0.107112471	0.059397735 <	0.126187210	1967	0.014000000	0.1111111111 <	0.1212359763
1968	0.091707273	**0.155163421 >**	0.104627261	1968	0.432692308	0.170000000 <	0.5842059706
1969	0.089483757	**0.156369930 >**	0.097133438	**1969**	0.117449664	**0.273504274 >**	0.2356260458
1970	0.113708893	**0.155658229 >**	0.130563942	1970	0.156505343	0.117449664 <	0.2788111432
1971	0.128951059	0.092234923 <	0.145911640	**1971**	0.004025765	**0.243243243 >**	0.1102055473
1972	0.133171674	**0.274836315 >**	0.150889331	1972	0.295825771	0.050724638 <	0.4328658957
1973	0.128327204	**0.175226464 >**	0.143780371	1973	0.377564979	0.094252874 <	0.5232490649
1974	0.011620503	**0.147482881 >**	0.010176734	**1974**	0.071534274	**0.558823529 >**	0.1848541082
1975	0.049283886	**0.271868534 >**	0.053163510	1975	0.308452776	0.270889488 <	0.4468274023
1976	0.138606148	0.143870983 <	0.152885964	**1976**	0.292779487	**0.590668081 >**	0.4294967868

1977	0.101896859	**0.261094507 >**	0.111211641	1977	0.476442274	0.355333333 <	0.6325826647
1978	0.135939160	**0.222974077 >**	-0.130110624	1978	0.267411938	0.272011805 <	0.4014464418
1979	0.081739434	**0.136440180 >**	0.100867603	**1979**	0.040450614	**0.244779582 >**	0.1504827962
1980	0.073012254	0.058777181 <	0.089224417	1980	0.116941735	0.078285182 <	0.2350630751
1981	0.061626927	**0.380613344 >**	0.076521832	**1981**	0.065927655	**0.234514549 >**	0.1786540662
1982	0.035512312	**0.216423434 >**	0.035240087	1982	0.066424682	0.015169195 <	0.1792036303
1983	0.084469087	**0.145530063 >**	0.099291801	1983	0.037831228	0.013103448 <	0.1475863001
1984	0.105996571	-0.056096026 <	0.137146334	1984	0.107484599	-0.016791468 <	0.2246058036
1985	0.049525101	**0.103005132 >**	0.056707013	1985	0.105172041	0.066466651 <	0.2220486850
1986	0.116370175	0.129620079 <	0.137396495	**1986**	-0.067712359	**0.105172041 >**	0.0308810222
1987	0.127448658	0.044422953 <	0.157953231	1987	0.208852271	0.021930683 <	0.3366934722
1988	0.078404261	0.040418331 <	0.093708566	1988	0.249736401	0.208852271 <	0.3819013746
1989	0.082324736	**0.152849121 >**	0.100388592	1989	0.090473333	0.008559201 <	0.2057955453
1990	0.053949764	-0.091632639 <	0.062451923	1990	0.185944118	0.043061449 <	0.3113627310
1991	0.075539582	0.082084047 <	0.086313290	1991	0.057551487	-0.029682085 <	0.1693920492
1992	0.074874594	0.045174743 <	0.084275142	1992	0.761070153	0.116304348 <	0.9473107612
1993	0.070138327	0.033468494 <	0.082005017	**1993**	0.160944108	**0.668382251 >**	0.2837189261
1994	0.071080235	-0.046310684 <	0.076838609	1994	0.078186351	0.064198766 <	0.1922091069
1995	0.064240461	-0.024095919 <	0.073199733	1995	0.048082147	0.110858665 <	0.1589212469
1996	0.061022509	0.024107901 <	0.070445851	**1996**	0.002363874	**0.140559983 >**	0.1083680717
1997	0.063667142	0.040602237 <	0.067732993	1997	-0.047236375	-0.051817957 <	0.0535223833
1998	0.043294441	0.022551187 <	0.048508201	1998	-0.034703761	-0.156123647 <	0.0673803737
1999	0.053191402	0.035529984 <	0.057850582	1999	0.107591945	-0.065842349 <	0.2247244474
2000	0.057811474	**0.416180218 >**	0.063874417	2000	0.010723431	0.033752482 <	0.1176116931
2001	-0.022240002	**-0.330831061 >**	-0.025733376	2001	0.260766291	0.010723431 <	0.3940976289

2002	0.039444946	-0.032263687 <	0.043448314	2002	0.071970566	0.035629454 <	0.1853359783
2003	0.033349860	-0.008641403 <	0.035702656	2003	0.120770966	0.118577982 <	0.2392972118
2004	0.056985754	0.020398302 <	0.061819889	2004	0.143460272	0.120770966 <	0.2643860083
2005	0.036580777	-0.018024434 <	0.039277152	2005	0.12660084	0.238748628 <	0.2457436203
2006	0.071874117	-0.026340865 <	0.077778525	2006	0.102463744	0.213262443 <	0.2190539232
2007	0.060261393	**0.210544041 >**	0.064996197	2007	0.020234692	0.078839521 <	0.1281288010
2008	-0.038697133	**0.092862193 >**	-0.043577266				

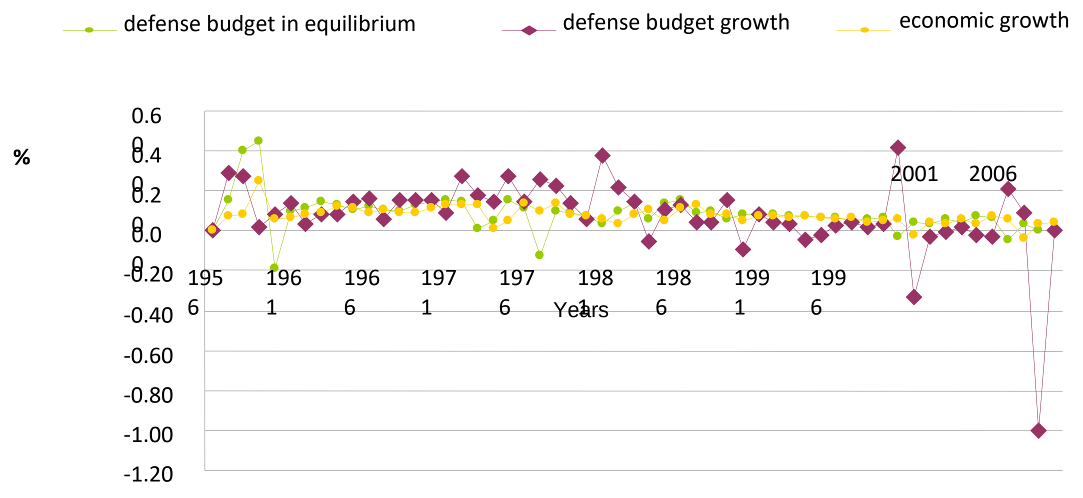

Figure3 TWN economic growth, defense budget growth and defense budget in equilibrium

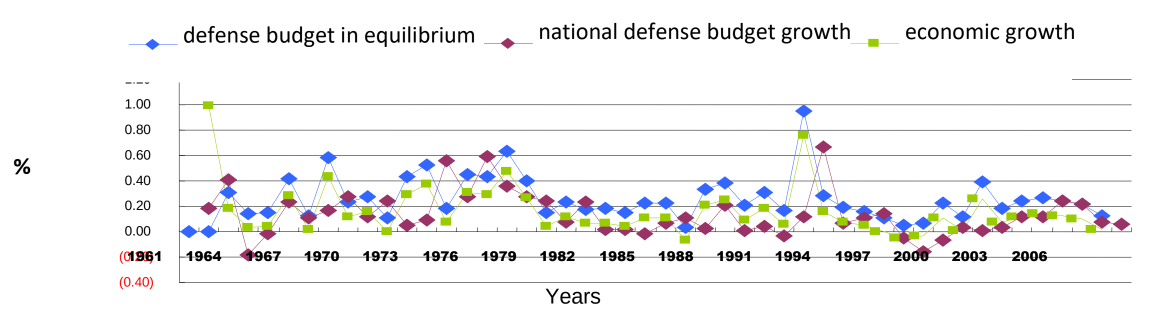

Figure 4 KOREA economic growth, defense budget growth and defense budget in equilibrium

5.4 Results

In the 52 years between 1957 and 2008, Taiwan's true national defense expenditure growth rate was greater than that of the equilibrium solution for a period of 23 years (see Table 12 and Figure 3). This suggests that Taiwan's actual national defense expenditure did not exceed the reasonable limit for social economic development. It is worth noting that Taiwan's economic growth from 1961 to 2000 has grown consistently.

This validates the fact that Taiwan's actual national defense expenditure from 1961 to 2000 has allowed the nation to construct solid and sound national defense security, which in turn, has supported the nation's economic growth. In addition, this finding also renders the notion that the increase in Taiwan's national defense budget has been detrimental to the country's economic development owing to "crowding out," as invalid.

For ten years, South Korea's actual national defense expenditure was higher than that of the equilibrium solution (see Table 12 and Figure 4). This shows that South Korea's DMU has been more effective than Taiwan's in terms of national defense resource utilization.

Overall, it is clear that, with the exception of 2001, Taiwan has enjoyed positive economic growth with a relatively high average growth rate of 13.90%. In the period between 1954 and 2005, South Korea witnessed an impressive average GDP growth rate of 21.14%. Just like Taiwan, South Korea's GDP has also shown steady growth since1987.

5.5 Descriptive statistics for input and output variables

From the process of model derivation, we have identified one input variable (annual national defense budget expenditure) and two output variables (defensive military strength and organizational structure ratio). As the sample set of seven DMUs was chosen and the research focused on a period that spanned 48 years, the analysis clearly adheres to the rule of thumb that the number of DMUs must be at least 200% greater than the sum of the input and output items (Galvin, 2003). In this paper, we have performed the Pearson correlation coefficient analysis on the I/O items used in the model to examine their isotonicity. An I/O item has to be removed if the coefficient turns out to be negative. However, as Table 14 makes clear, all the relevant coefficients in our analysis are positive. This means that the set of I/O items are adequately isotonic.

TABLE 14 Table of descriptive statistics for input and output variables

	(Input) National defense budget	(Output) Defensive military strength	(Output) Military strength organizational structure ratio
Max.	4029.67	18.889	0.617
Min.	1.10	0.166	0.051
Mean	523.5509	5.23016	0.24297
Std	926.71852	4.989647	0.117053

TABLE 15 Table of descriptive statistics for input and output variables

	National defense budget	Defensive military strength	Military strength organizational structure ratio
National defense budget	1	0.491	0.569
Defensive military strength	0.491	1	0.450
Military strength organizational structure ratio	0.568	0.450	1

5.6 Analysis on changes in productivity

In this paper, we analyze the changes of TE for the subjects. Change in TE can be categorized even further into change in pure technical efficiency (PTE) and change in scale efficiency (SE). After covering these changes, we analyze the change in technical reform, before moving on to conduct an analysis of TFP change (Malmquist Productivity Index) to attempt to identify the causes that led to TFP decline.

5.7 Analysis on changes in efficiency

5.7.1 Technical efficiency

The changes in PTE and SE reflect the changes that have taken place in the subject DMUs during the time span examined. If a subject's overall efficiency = 1, then the subject should be regarded as relatively efficient. If the value is smaller than 1, then the subject in question should be regarded as relatively inefficient. In terms of the TE of national defense expenditure for Taiwan, Taiwan's equilibrium solution, South Korea, South Korea's equilibrium solution, Japan, India, and Israel, It was India that was found to have the most efficient TE performance (1.042), followed by Japan (1.032), and Israel (1.000). These three countries showed improvement in their efficiency during this time period, while the remaining DMUs showed decline. The technical inefficiency of Taiwan and its equilibrium solution stems from its insufficient TE, while both TE and PTE meant that South Korea's equilibrium solution were inefficient. (see Table 16).

TABLE 16 Various efficiency changes

DMU	TE	TC	PTE	SE	TFP
TWN	0.979	0.928	1.005*	0.973	0.908
TWN-E	0.973	0.928	1.005*	0.973	0.903
KOR	0.973	0.931	0.977	0.995	0.905
KOR-E	0.982	0.931	0.986	0.995	0.914
JAPAN	1.032*	0.921	1.007*	1.025*	0.951
INDIA	1.042*	0.921	1.037*	1.005*	0.960
ISRAEL	1.000*	0.923	1.000*	1.000*	0.923
Mean	0.997	0.926	1.002*	0.995	0.923

5.7.2 The change in catch-up effects

TE reflects the relationship between input production factors and mass production (i.e., how to maximize mass production under given production factors or minimize input production factors under specific mass production target). In other words, the value of TE reflects the correlation between overall efficiency and management decisions. We can use TE to assess whether the DMUs in our analysis were able to achieve maximum output with minimal input to produce maximum output (a value of 1 denotes efficiency). When a DMU is found to be able to decrease input while maintaining the same level of production, this indicates that the DMU's technology has yet to reach optimal state and its efficiency will be less than 1.

As Table 17 shows, the average TE for all seven DMUs is greater than 1, which means the DMUs in our analysis were able to enhance their military capabilities through the effective management and utilization of national defense resources without squandering the national treasury or making inappropriate investments.

TABLE 17 TE changes for national defense expenditure (1961-2008)

Catch-up	TWN	KOR	TWN-E	KOR-E	JAPAN	INDIA	ISRAEL	Average
1961=>1962	0.97073567	0.96451525	0.65341619	0.96095864	6.58770169*	1.06603974*	0.87775156	1.72587410*
1962=>1963	1.07485051*	0.85879329	1.09903420*	1.58302336*	1.26220587*	1.29779026*	0.91124460	1.15527744*
1963=>1964	0.93689798	1.41035933*	0.91239840	0.73011564	1.00040798*	0.96804810	0.85685506	0.97358321
1964=>1965	0.74001799	1.06053419*	0.79697945	1.00487981*	1.02609537*	1.05682798*	0.80274299	0.92686825
1965=>1966	1.16204726*	1.27336369*	1.20562639*	0.78532159	1.25929615*	4.11131833*	0.61290903	1.48712606*
1966=>1967	0.98498060	0.82130370	0.87881497	1.21757640*	0.94388876	0.92928270	1.02879173*	0.97209126
1967=>1968	0.96504766	1.12650702*	1.07267696*	0.88769975	0.80212019	0.79546467	1.82256783*	1.06744058*
1968=>1969	1.15761109*	0.76177119	1.16660418*	1.31273014*	1.22685901*	1.33184928*	0.82104631	1.11121017*
1969=>1970	0.87503181	1.43592015*	0.84878083	0.69641755	1.37394967*	2.89548532*	0.55936400	1.24070705*
1970=>1971	1.19631832*	0.89845883	1.11550773*	1.11301705*	1.05643499*	0.93874560	0.87350205	1.02742637*
1971=>1972	0.76255031	0.81007035	0.88618980	1.23446068*	0.25171431	0.35553806	4.49485920*	1.25648325*
1972=>1973	0.93853442	1.73952672*	0.87054010	0.57486901	0.78272230	0.80577735	1.00341190*	0.95934026
1973=>197	1.3440977	0.8055217	1.4854818	1.2414313	2.8410115	2.8363666	0.5369889	1.5844142

4	3*	9	5*	5*	0*	1*	8	6*
1974=>1975	1.00533551*	1.42943291*	1.06894097*	0.69957813	0.81264913	0.86207151	1.35062703*	1.03266217*
1975=>1976	1.38974936*	0.57241510	1.14197688*	1.36720023*	2.33313482*	2.17734957*	0.38884840	1.33866777*
1976=>1977	0.93779505	0.72339509	1.07258600*	1.29793025*	1.20065866*	0.97371353	1.29679282*	1.07183877*
1977=>1978	0.65313563	0.72507887	0.80300095	0.74219207	0.64216282	1.14668448*	1.62304186*	0.90504238
1978=>1979	0.48499178	0.49425958	0.35903846	0.59788367	0.39854787	0.44677930	2.66358340*	0.77786915
1979=>1980	2.40658857*	2.54226555*	2.26571707*	2.21950377*	2.95099576*	2.37669440*	0.35458156	2.15947810*
1980=>1981	1.03758150*	1.04834100*	1.36882720*	1.27543577*	1.05050866*	1.01562295*	0.88548987	1.09740099*
1981=>1982	0.69025955	0.82081470	0.63208947	0.79163323	0.89906218	0.88664742	1.24389198*	0.85205693
1982=>1983	0.94154575	1.06575295*	0.83541075	1.10045554*	0.94806361	1.03667770*	0.90849662	0.97662899
1983=>1984	1.00149872*	0.63635989	0.79807879	0.69547436	0.61510537	0.53489090	1.11443036*	0.77083406
1984=>1985	0.62281373	0.67508992	0.78266300	0.50240705	0.66623599	0.67881959	1.45774241*	0.76939595
1985=>1986	1.19691907*	1.48073973*	1.13920710*	1.54896626*	1.00610620*	1.40996861*	0.76831934	1.22146090*
1986=>1987	0.96137063	1.04790262*	0.87323130	1.01524886*	0.74781708	0.70408064	1.12857225*	0.92546048
1987=>1988	1.25613253*	1.02539073*	1.32435557*	1.13008748*	0.95186558	1.05094602*	0.93219477	1.09585324*
1988=>1989	0.79127529	1.41182827*	0.87149846	1.53086212*	1.10485449*	1.23085989*	1.03391784*	1.13929948*
1989=>1990	0.99661402	0.69754010	0.81313714	0.45028750	1.08289423*	0.95398375	0.77096325	0.82363142
1990=>1991	0.90074662	1.00449917*	1.04947828*	1.32448284*	0.81949137	0.99480027	0.99552098	1.01271707*
1991=>1992	0.84201152	1.40598719*	0.81479390	1.01294515*	1.14859643*	1.59272791*	1.49586748*	1.18756137*
1992=>1993	1.16841789*	0.60994189	1.15780898*	0.98369346	0.88743426	1.08513579*	1.65829112*	1.07867477*
1993=>1994	1.08597633*	1.04100809*	1.00682774*	1.04276866*	1.04137749*	0.96063345	0.98801181	1.02380051*
1994=>1995	1.08224856*	0.86269601	1.11128202*	0.88397240	0.72939913	0.95184989	1.15915687*	0.96865784
1995=>1996	0.94717308	0.78972483	0.99652227	0.93761975	1.20470451*	0.94910890	1.09708065*	0.98884771
1996=>1997	0.91711834	1.01362069*	0.93415936	0.99258842	0.98019831	0.80000000	0.98768963	0.94648211
1997=>1998	0.96733405	1.25682638*	0.96800550	1.24417474*	1.30798708*	1.22144071*	0.79565485	1.10877476*
1998=>1999	0.92611024	0.95744232	0.92957096	0.69360781	0.78291245	0.80538594	0.96762001	0.86609282
1999=>2000	0.73534760	1.09476944*	0.99998558	1.31691765*	0.93300795	0.69013632	0.96932150	0.96278372
2000=>2001	1.77925021*	1.17626219*	0.91787834	1.09798921*	1.37268207*	1.01174451*	0.74401243	1.15711699*
2001=>2002	1.07926553*	1.00851172*	1.45754395*	0.91519759	0.98805425	1.03774929*	0.99156012	1.06826892*

2002=>2003	0.92843373	0.73667090	0.95817902	0.74623209	0.85133888	1.04407513*	1.45652740*	0.96020816
2003=>2004	0.63650368	0.93514478	0.63906354	0.86460610	0.96101174	0.87520343	1.06446853*	0.85371454
2004=>2005	1.23490807*	0.73750288	1.21413738*	0.81368513	1.01260238*	0.88401196	1.22130727*	1.01687930*
2005=>2006	0.94222999	0.75725155	0.90092810	1.03153851*	0.92878777	0.85885269	1.33364853*	0.96474816
2006=>2007	1.12868265*	1.26776245*	1.42009644*	1.21815964*	1.38530309*	0.92718298	0.85930448	1.17235596*
2007=>2008	0.92955271	0.96124530	0.89365171	1.00156197*	0.97426508	1.17632296*	1.04031718*	0.99670242
Average	1.01518380*	1.02085362*	1.01088773*	1.03041252*	1.19434520*	1.16469546*	1.12657212*	1.08042149*

5.7.3 Change in pure technical efficiency (PTE)

Overall technical inefficiency may be a result of inefficiency in PTE or SE. PTE is an indicator of DMUs' ability to effectively utilize relevant input items to achieve output maximization or input minimization; it reflects input efficiency. As Table 17 makes clear, Taiwan (1.005), Taiwan's equilibrium solution (1.005), Japan (1.007), India (1.037), and Israel (1.000) have all reached optimal PTE. In order to improve their efficiency, it is clear that South Korea's DMU should gradually revise its methods of national defense resource management and utilization.

TABLE 18 PTE changes for defense expenditure (1961-2008)

PTE	TWN	KOR	TWN-E	KOR-E	JAPAN	INDIA	ISRAEL	Average
1961=>1962	1.408*	1.000*	0.948	0.957	1.410*	1.094*	1.000*	1.102*
1962=>1963	1.000*	0.944	1.022*	1.615*	1.000*	1.241*	1.000*	1.099*
1963=>1964	1.000*	1.059*	0.974	0.714	1.000*	0.802	1.000*	0.928
1964=>1965	0.984	1.000*	1.059*	1.050*	1.000*	0.938	1.000*	1.004*
1965=>1966	0.964	1.000*	1.000*	0.750	1.000*	2.554*	1.000*	1.092*
1966=>1967	1.054*	1.000*	0.941	1.226*	1.000*	0.850	1.000*	1.005*
1967=>1968	0.956	1.000*	1.063*	0.878	1.000*	1.185*	1.000*	1.008*
1968=>1969	0.993	1.000*	1.000*	1.340*	1.000*	1.096*	1.000*	1.055*
1969=>1970	1.031*	1.000*	1.000*	0.690	1.000*	2.347*	1.000*	1.076*
1970=>1971	1.022*	1.000*	0.954	1.111*	1.000*	1.011*	1.000*	1.013*
1971=>1972	0.903	1.000*	1.048*	1.265*	0.313	0.625	1.000*	0.813
1972=>1973	1.078*	1.000*	1.000*	0.571	0.789	0.824	1.000*	0.877
1973=>1974	0.905	1.000*	1.000*	1.222*	2.533*	1.949*	1.000*	1.274*
1974=>1975	0.941	1.000*	1.000*	0.700	0.840	1.024*	1.000*	0.922
1975=>1976	1.208*	1.000*	0.992	1.372*	1.906*	1.584*	1.000*	1.257*
1976=>1977	0.881	1.000*	1.008*	1.294*	1.000*	0.990	1.000*	1.019*
1977=>1978	0.813	1.000*	1.000*	0.955	1.000*	1.074*	1.000*	0.974
1978=>1979	1.351*	0.503	1.000*	0.609	1.000*	0.527	1.000*	0.804
1979=>1980	1.033*	1.988*	0.973	1.735*	1.000*	1.936*	1.000*	1.312*
1980=>1981	0.779	1.000*	1.028*	1.251*	1.000*	0.994	1.000*	1.000*
1981=>1982	1.092*	1.000*	1.000*	0.937	1.000*	0.987	1.000*	1.001*
1982=>1983	1.127*	1.000*	1.000*	1.033*	1.000*	1.009*	1.000*	1.023*
1983=>1984	1.043*	1.000*	0.831	1.093*	1.000*	0.827	1.000*	0.966
1984=>1985	0.958	0.784	1.204*	0.583	1.000*	0.856	1.000*	0.892
1985=>1986	1.044*	1.276*	0.994	1.337*	1.000*	1.243*	1.000*	1.119*
1986=>1987	1.000*	0.979	0.908	0.947	1.000*	0.775	1.000*	0.941
1987=>1988	1.000*	0.924	1.054*	1.020*	1.000*	1.100*	1.000*	1.013*
1988=>1989	0.954	1.105*	1.051*	1.197*	1.000*	1.209*	1.000*	1.070*
1989=>1990	1.048*	0.952	0.855	0.615	1.000*	0.927	1.000*	0.902
1990=>1991	1.000*	1.005*	1.165*	1.324*	1.000*	0.979	1.000*	1.061*
1991=>1992	1.000*	0.983	0.968	0.708	1.000*	1.455*	1.000*	0.997

1992=>1993	1.000*	0.599	0.991	0.965	1.000*	1.000*	1.000*	0.923
1993=>1994	1.000*	1.003*	0.927	1.004*	1.000*	0.986	1.000*	0.988
1994=>1995	1.000*	0.862	1.027*	0.884	1.000*	0.862	1.000*	0.945
1995=>1996	1.000*	0.890	1.052*	1.056*	1.000*	0.981	1.000*	0.996
1996=>1997	1.000*	0.995	1.019*	0.974	1.000*	0.800	1.000*	0.967
1997=>1998	1.000*	1.256*	1.001*	1.244*	1.000*	1.061*	1.000*	1.075*
1998=>1999	1.000*	1.024*	1.004*	0.742	1.000*	0.869	1.000*	0.943
1999=>2000	0.751	1.011*	1.021*	1.216*	1.000*	0.716	1.000*	0.945
2000=>2001	1.331*	1.272*	0.687	1.187*	1.000*	1.319*	1.000*	1.089*
2001=>2002	1.000*	1.009*	1.350*	0.915	1.000*	1.038*	1.000*	1.037*
2002=>2003	1.000*	0.704	1.032*	0.713	1.000*	0.758	1.000*	0.875
2003=>2004	1.000*	0.941	1.004*	0.871	1.000*	0.881	1.000*	0.955
2004=>2005	1.000*	0.814	0.983	0.897	1.000*	0.889	1.000*	0.938
2005=>2006	1.000*	0.751	0.956	1.023*	1.000*	0.853	1.000*	0.935
2006=>2007	0.880	1.222*	1.107*	1.174*	1.000*	1.087*	1.000*	1.061*
2007=>2008	1.040*	0.961	1.000*	1.001*	1.000*	1.144*	1.000*	1.020*
Average	1.005*	0.977	1.005*	0.986	1.007*	1.037*	1.000*	

5.7.4 Change in scale efficiency (SE)

SE reveals the ratio between output and input. In order to achieve scale efficiency, a DMU has to achieve proportionate growth. If growth is not proportional between input and output, the DMU is regarded as inefficient in terms of its scale of operation. A high SE value indicates scale fitness and high productivity. When the value of SE is smaller than 1, this indicates that an operation is increasing returns to scale, and the DMU should enlarge its scale of production. If SE = 1, this indicates that an operation has reached constant returns to scale (i.e., ideal scale of production) and the DMU could either expand or downsize the scale of production. However, when SE exceeds 1, the operation is then clearly decreasing returns to scale and the DMU should reduce the scale of production.

The results of our analysis are provided in Table 19, from which we can see that the subject DMUs that have reached the optimal average SE are Japan (1.025), India (1.005), and Israel (1.000). The average SE for Taiwan and its equilibrium solution was 0.973. The average SE for South Korea and its equilibrium solution, was 0.995, indicating that the SEs for both countries still have room for improvement (2.7% and 0.05% respectively).

TABLE 19 SE changes for defense expenditure(1961-2008)

SE	TWN	KOR	TWN-E	KOR-E	JAPAN	INDIA	ISRAEL	Average
1961=>1962	0.687	1.000*	0.687	1.000*	4.670*	0.882	1.000*	1.100*
1962=>1963	1.076*	0.995	1.076*	0.995	1.242*	1.129*	1.000*	1.070*
1963=>1964	0.942	1.005*	0.942	1.005*	0.994	1.152*	1.000*	1.004*
1964=>1965	0.767	1.000*	0.767	1.000*	1.031*	1.125*	1.000*	0.947
1965=>1966	1.174*	1.000*	1.174*	1.000*	1.347*	1.713*	0.857	1.154*
1966=>1967	0.936	1.000*	0.936	1.000*	0.863	1.038*	0.951	0.959
1967=>1968	0.991	1.000*	0.991	1.000*	0.823	0.676	1.227*	0.945
1968=>1969	1.194*	1.000*	1.194*	1.000*	1.227*	1.170*	1.000*	1.108*
1969=>1970	0.836	1.000*	0.836	1.000*	1.307*	1.202*	0.655	0.954
1970=>1971	1.170*	1.000*	1.170*	1.000*	1.081*	0.993	1.152*	1.078*
1971=>1972	0.861	1.000*	0.861	1.000*	0.856	0.598	1.325*	0.906
1972=>1973	0.869	1.000*	0.869	1.000*	0.966	0.944	1.000*	0.948
1973=>1974	1.470*	1.000*	1.470*	1.000*	1.149*	1.506*	1.000*	1.207*
1974=>1975	1.068*	1.000*	1.038*	1.000*	0.951	0.835	1.000*	0.986
1975=>1976	1.156*	1.000*	1.156*	1.000*	1.106*	1.386*	0.813	1.076*
1976=>1977	1.063*	1.000*	1.063*	1.000*	1.000*	0.924	1.230*	1.036*
1977=>1978	0.803	0.799	0.803	0.799	0.845	1.062*	1.000*	0.867
1978=>1979	0.359	0.957	0.359	0.957	0.403	0.839	1.000*	0.631
1979=>1980	2.328*	1.253*	2.328*	1.253*	2.938*	1.230*	1.000*	1.631*
1980=>1981	1.332*	1.044*	1.332*	1.044*	1.000*	1.035*	1.000*	1.104*
1981=>1982	0.632	0.887	0.632	0.887	0.984	0.910	1.000*	0.834

1982=>1983	0.836	0.976	0.836	0.976	0.897	1.000*	1.000*	0.929
1983=>1984	0.960	0.661	0.960	0.661	0.644	0.668	1.000*	0.778
1984=>1985	0.650	0.861	0.650	0.861	0.646	0.813	1.000*	0.773
1985=>1986	1.148*	1.000*	1.148*	1.111*	1.008*	1.137*	1.000*	1.093*
1986=>1987	0.961	1.124*	0.961	1.124*	0.773	0.913	1.000*	0.973
1987=>1988	1.256*	1.125*	1.256*	1.125*	0.933	0.923	1.000*	1.081*
1988=>1989	0.829	1.280*	0.829	1.280*	1.101*	1.002*	1.000*	1.031*
1989=>1990	0.951	0.738	0.951	0.738	1.088*	1.082*	1.000*	0.925
1990=>1991	0.901	1.000*	0.901	1.000*	0.833	1.000*	1.000*	0.946
1991=>1992	0.840	1.402*	0.840	1.402*	1.140*	1.085*	1.000*	1.080*
1992=>1993	1.172*	1.013*	1.172*	1.013*	0.874	1.079*	1.000*	1.042*
1993=>1994	1.087*	1.029*	1.087*	1.029*	1.036*	0.966	1.000*	1.033*
1994=>1995	1.063*	1.000*	1.063*	1.000*	0.728	1.077*	1.000*	0.983
1995=>1996	0.991	0.909	0.991	0.909	1.235*	1.000*	1.000*	1.000*
1996=>1997	0.906	1.000*	0.906	1.000*	0.991	1.000*	1.000*	0.971
1997=>1998	0.953	1.000*	0.953	1.000*	1.305*	1.214*	1.000*	1.053*
1998=>1999	0.926	0.920	0.926	0.920	0.775	0.866	1.000*	0.902
1999=>2000	0.960	1.100*	0.960	1.100*	0.917	1.000*	1.000*	1.003*
2000=>2001	1.358*	0.926	1.358*	0.926	1.347*	0.752	1.000*	1.070*
2001=>2002	1.079*	1.000*	1.079*	1.000*	0.988	1.000*	1.000*	1.020*
2002=>2003	0.942	1.080*	0.942	1.080*	0.899	1.412*	1.000*	1.040*
2003=>2004	0.622	0.962	0.622	0.962	0.931	0.962	1.000*	0.850
2004=>2005	2.231*	0.909	1.231*	0.909	1.018*	1.000*	1.000*	1.035*
2005=>2006	0.936	1.000*	0.936	1.000*	0.922	1.000*	1.000*	0.970
2006=>2007	1.287*	1.040*	1.287*	1.040*	1.388*	0.856	1.000*	1.114*
2007=>2008	0.894	1.000*	0.894	1.000*	0.962	1.000*	1.000*	0.963
Average	0.973	0.995	0.973	0.995	1.025*	1.005*	1.000*	0.995

5.7.5 Change in frontier-shift effects

Technical changes refer to those changes in production frontiers that take place over time. If the value of technical change is greater than 1, this indicates that the entire sector has shown improvement; if the value = 1, the entire sector remains unchanged. If the value is smaller than 1, the entire sector has shown decline. The findings that emerge from our analysis (see Table 20) show that none of the subject DMUs achieved the optimal technical changes of the average production frontier. The means for each subject DMU are as follows: Taiwan (0.95727), South Korea (0.962236), Taiwan's equilibrium solution (0.95727), South Korea's equilibrium solution (0.96062), Japan (0.97883), India (0.9766), and Israel (0.9283). These results imply that all of our DMUs could improve in this area.

TABLE 20 Technical changes for defense expenditure(1961 – 2008)

Frontier	TWN	KOR	TWN-E	KOR-E	JAPAN	INDIA	ISRAEL	Average
1961=>1962	0.853493902	0.864313511	0.853493902	0.848840027	0.813157895	0.813157895	0.897449597	0.849129532
1962=>1963	0.811823452	0.830496641	0.811823452	0.800573278	0.79334258	0.79334258	0.862120174	0.814788879
1963=>1964	1.086202388*	0.951403519	1.086202388*	1.086202388*	0.833333333	0.833333333	0.900254022	0.968133053
1964=>1965	1.030252145*	1.012898551*	1.030252145*	1.043105624*	0.885608856	0.885608856	0.988447687	0.982310552
1965=>1966	0.767601177	0.722173446	0.767601177	0.814925199	0.705654339	0.705654339	0.833735544	0.759620746
1966=>1967	0.963610917	1.049297168*	0.963610917	0.950934033	0.938423921	0.938423921	0.938423921	0.963246399
1967=>1968	0.865873415	0.771444373	0.865873415	0.818788184	1.140398428*	1.140398428*	0.808106708	0.915840422
1968=>1969	0.785234899	0.943827812	0.785234899	0.823768237	0.757539204	0.757539204	0.836028892	0.812739021
1969=>19	0.9309793	0.7059640	0.9309793	0.8459559	0.5894774	0.5894774	0.7151767	0.7582871

70	66	75	66	45	27	27	58	95
1970=>1971	0.790942029	0.840973154	0.790942029	0.797133799	0.825103412	0.847334587	0.829298028	0.817389577
1971=>1972	0.952474413	1.063511439*	0.952474413	0.957201862	2.633882701*	2.550781809*	1.175864627*	1.469455895*
1972=>1973	0.91206589	0.697685325	0.91206589	0.92018498	0.969077936	0.968840632	0.966414815	0.906619352
1973=>1974	0.632901967	0.711088201	0.632901967	0.638207817	0.360340084	0.362981375	0.49378287	0.547457755
1974=>1975	0.786850477	0.658127934	0.786850477	0.786850477	1.065365276*	1.065365276*	0.916754481	0.866594914
1975=>1976	0.598490667	0.850237994	0.598490667	0.643274074	0.408687131	0.431644893	0.7335307	0.609193732
1976=>1977	0.827569363	0.882183666	0.827569363	0.750320464	0.757581472	0.826725112	0.807249012	0.811314065
1977=>1978	1.290030211*	1.150985938*	1.290030211*	1.109939544*	0.969691954	0.842765877	1.012593234*	1.095148138*
1978=>1979	2.063143631*	2.14798987*	2.063143631*	2.14798987*	2.14798987*	2.14798987*	1.289875212*	2.001160279*
1979=>1980	0.319063545	0.383421914	0.319063545	0.383421914	0.38421686	0.386995168	0.585783463	0.39456663
1980=>1981	0.718472058	0.790908727	0.718472058	0.801546967	0.826942641	0.846155551	0.778300388	0.782971199
1981=>1982	1.225305089*	1.184169407*	1.225305089*	1.168452951*	1.131483344*	1.097825371*	1.111539962*	1.163440173*
1982=>1983	0.927154439	0.900182989	0.927154439	0.900182989	0.900182989	0.900182989	0.958472777	0.91621623
1983=>1984	1.103441134*	1.598275199*	1.103441134*	1.598275199*	1.641979905*	1.641979905*	1.275064155*	1.42320809*
1984=>1985	1.335370871*	1.336513456*	1.335370871*	1.336513456*	1.352213625*	1.352213625*	1.112969362*	1.308737895*
1985=>1986	0.706294292	0.627997975	0.706294292	0.627997975	0.624608624	0.624608624	0.757750017	0.667935971
1986=>1987	0.995939006	1.112159443*	0.995939006	1.112159443*	1.112159443*	1.112159443*	0.990684149	1.06159999*
1987=>1988	0.894921191	0.894921191	0.894921191	0.894921191	0.894921191	0.894921191	0.926896965	0.899489159
1988=>1989	0.896389325	0.896389325	0.896389325	0.896389325	0.896389325	0.896389325	0.881563625	0.894271368
1989=>1990	1.007911392*	1.007911392*	1.007911392*	1.007911392*	1.007911392*	1.007911392*	1.147903861*	1.027910317*
1990=>1991	1.025974026*	1.025974026*	1.025974026*	1.025974026*	1.025974026*	1.025974026*	1.028279454*	1.026303373*
1991=>1992	1.118210058*	0.83341756	1.118210058*	0.83341756	0.83341756	0.83341756	0.793800263	0.909127231
1992=>1993	0.988304094	0.982688729	0.988304094	0.982688729	0.982688729	0.982688729	0.767467878	0.953547283
1993=>1994	0.928409091	0.902657807	0.928409091	0.902657807	0.902657807	0.902657807	0.920979993	0.912632772
1994=>1995	1.043478261*	1.043478261*	1.043478261*	1.043478261*	1.043478261*	1.043478261*	0.969197532	1.032866728*
1995=>1996	1.018499215*	1.018499215*	1.018499215*	1.018499215*	1.018499215*	1.018499215*	0.964920197	1.01084507*
1996=>1997	1.060606061*	1.060606061*	1.060606061*	1.060606061*	1.060606061*	1.060606061*	1.067195187*	1.061547364*
1997=>1998	0.942857143	0.942857143	0.942857143	0.942857143	0.942857143	0.942857143	1.057020814*	0.959166239

1998=>1999	1.07615775*	1.118065398*	1.07615775*	1.118065398*	1.118065398*	1.118065398*	1.094015603*	1.102656099*
1999=>2000	0.940560672	0.940560672	0.940560672	0.940560672	0.940560672	0.940560672	0.960932044	0.943470868
2000=>2001	0.839899262	0.839899262	0.839899262	0.839899262	0.839899262	0.839899262	0.912253624	0.8502356
2001=>2002	0.957446809	0.957446809	0.957446809	0.957446809	0.957446809	0.957446809	0.961512935	0.958027684
2002=>2003	1.213557109*	1.213557109*	1.213557109*	1.213557109*	1.213557109*	1.213557109*	1.041590418*	1.188990439*
2003=>2004	0.954122802	0.954122802	0.954122802	0.954122802	0.954122802	0.954122802	0.922664267	0.949628726
2004=>2005	0.997518383	0.997518383	0.997518383	0.997518383	0.997518383	0.997518383	0.901101164	0.983744495
2005=>2006	1.09002419*	1.09002419*	1.09002419*	1.09002419*	1.09002419*	1.09002419*	0.947163954	1.069615585*
2006=>2007	0.731892935	0.731892935	0.731892935	0.731892935	0.731892935	0.731892935	0.823235727	0.744941905
2007=>2008	0.984375	0.984375	0.984375	0.984375	0.984375	0.984375	0.965111945	0.981623135
Average	0.957270117	0.962236064	0.957270117	0.960629999	0.978836309	0.976603178	0.928308043	0.960164832

5.7.6 Change in total factor productivity (TFP)

The Malmquist index is a tool used to assess cross-sectional TFP changes. $M^t > 1$ indicates improvement in productivity and $M^t < 1$ indicates decline in productivity. In their study, Fare et al. (1994) broke the Malmquist index (TFP) down further into technical efficiency (TE) and technical change (TC). Using this formula, the total factor productivity (TFP) can be represented as the ratio of total output against total input.

As Table 21 makes clear, only Israel and Japan showed improvement over the time period in terms of TFP; TFPs for the remaining five DMUs in the study are all smaller than 1 (indicating decline). The geometric mean for TFP change for all seven DMUs was 0.9716364, indicating that their mean TFP over the past 47 years has regressed slightly.

TABLE 21 TFP changes for defense expenditure from(1961- 2008)

Malmquist	TWN	KOR	TWN-E	KOR-E	JAPAN	INDIA	ISRAEL	Average
1961=>1962	0.82851697	0.83364356	0.55768673	0.81570016	5.35684164*	0.86685863	0.78773778	1.43528364
1962=>1963	0.87258885	0.71322494	0.89222174	1.26732620*	1.00136166*	1.02959228*	0.78560235	0.93741686
1963=>1964	1.01766082*	1.34182083*	0.99104932	0.79305335	0.83367332	0.80670675	0.77138722	0.93647880
1964=>1965	0.76240512	1.07421354*	0.82108978	1.04819579*	0.90871915	0.93593622	0.79346945	0.90628986
1965=>1966	0.89198885	0.91958945	0.92544024	0.63997835	0.88862779	2.90116962*	0.51100404	1.09682833
1966=>1967	0.94913806	0.86179165	0.84683570	1.15783483*	0.88576779	0.87206111	0.96544276	0.93412456
1967=>1968	0.83560911	0.86903750	0.92880246	0.72683807	0.91473661	0.90714666	1.47282929*	0.95071424
1968=>1969	0.90899662	0.71898084	0.91605832	1.08138539*	0.92939380	1.00892805*	0.68641844	0.89288021
1969=>1970	0.81463656	1.01370804*	0.79019744	0.58913857	0.80991231	1.70682323*	0.40004413	0.87492290
1970=>1	0.946218	0.755579	0.882301	0.887223	0.871668	0.795431	0.724393	0.837545

971	44	76	94	51	12	61	53	27
1971=>1972	0.72630966	0.86151909	0.84407311	1.18162806*	0.66298598	0.90690002	5.28534594*	1.49553741
1972=>1973	0.85600523	1.21364226*	0.79398993	0.52898583	0.75851892	0.78066984	0.96971213	0.84307488
1973=>1974	0.85068210	0.57279704	0.94016439	0.79229119	1.02373032*	1.02954825*	0.26515596	0.78205275
1974=>1975	0.79104872	0.94074973	0.84109671	0.55046339	0.86576816	0.91842105	1.23819338*	0.87796302
1975=>1976	0.83175202	0.48668907	0.68346250	0.87948446	0.95352217	0.93984182	0.28523224	0.72285490
1976=>1977	0.77609045	0.63816733	0.88763931	0.97386363	0.90959675	0.80499343	1.04683472*	0.86245509
1977=>1978	0.84256470	0.83455558	1.03589548*	0.82378833	0.62270011	0.96638655	1.64348120*	0.96705314
1978=>1979	1.00060771*	1.06166457*	0.74074791	1.28424807*	0.85607679	0.95967742	3.43569020*	1.33410181
1979=>1980	0.76785468	0.97476032	0.72290772	0.85100638	1.13382232*	0.91976925	0.20770801	0.79683267
1980=>1981	0.74547331	0.82914205	0.98346409	1.02232167*	0.86871041	0.85937500	0.68917711	0.85680909
1981=>1982	0.84577854	0.97198366	0.77450244	0.92498619	1.01727388*	0.97338403	1.38263564*	0.98436348
1982=>1983	0.87295832	0.95937267	0.77455478	0.99061136	0.85343073	0.93319963	0.87076928	0.89355668
1983=>1984	1.10509488*	1.01707824*	0.88063296	1.11155942*	1.00999066*	0.87828011	1.42097020*	1.06051521
1984=>1985	0.83168731	0.90226676	1.04514538*	0.67147379	0.90089338	0.91790909	1.62242264*	0.98454262
1985=>1986	0.84537711	0.92990155	0.80461547	0.97274767	0.62842261	0.88067855	0.58219399	0.80627671
1986=>1987	0.95746651	1.16543480*	0.86968511	1.12911860*	0.83169182	0.78304994	1.11805864*	0.97921506
1987=>1988	1.12413962*	0.91764390	1.18519386*	1.01133923*	0.85184468	0.94051386	0.86404851	0.98496052
1988=>1989	0.70929072	1.26554779*	0.78120192	1.37224846*	0.99037977	1.10332967*	0.91146436	1.01906610
1989=>1990	1.00449862*	0.70305861	0.81957019	0.45384991	1.09146143*	0.96153109	0.88499169	0.84556593
1990=>1991	0.92414263	1.03059006*	1.07673745*	1.35888500*	0.84077686	1.02063923*	1.02367377*	1.03934929
1991=>1992	0.94154575	1.17177441*	0.91111073	0.84420627	0.95726043	1.32740741*	1.18742000*	1.04867500
1992=>1993	1.15475218*	0.59938302	1.14426735*	0.96666448	0.87207164	1.06635071*	1.27268516*	1.01088208
1993=>1994	1.00823030*	0.93967408	0.93474803	0.94126327	0.94000752	0.86712329	0.90993911	0.93442651
1994=>1995	1.12930285*	0.90020453	1.15959863*	0.92240598	0.76111213	0.99323467	1.12345197*	0.99847297
1995=>1996	0.96469504	0.80433411	1.01495715*	0.95496498	1.22699060*	0.96666667	1.05859528*	0.99874340
1996=>1997	0.97270127	1.07505225*	0.99077508	1.05274529*	1.03960427*	0.84848485	1.05405762*	1.00477438
1997=>1998	0.91205782	1.18500773*	0.91269090	1.17307904*	1.23324496*	1.15164410*	0.84102373	1.05839261
1998=>1999	0.99664071	1.07048312*	1.00036499*	0.77549889	0.87534732	0.90047415	1.05859139*	0.95391437

1999=>2000	0.69163903	1.02969708*	0.94054711	1.23864095*	0.87755059	0.64911508	0.93145209	0.90837742
2000=>2001	1.49439094*	0.98794175	0.77092534	0.92220033	1.15291466*	0.84976347	0.67872804	0.97955207
2001=>2002	1.03333934*	0.96559633	1.39552080*	0.87625301	0.94600939	0.99358974	0.95339788	1.02338664
2002=>2003	1.12670735*	0.89399221	1.16280496*	0.90559525	1.03314836*	1.26704480*	1.51710498*	1.12948541
2003=>2004	0.60730267	0.89224296	0.60974510	0.82494039	0.91692321	0.83505155	0.98214707	0.80976471
2004=>2005	1.23184350*	0.73567268	1.21112436*	0.81166588	1.01008949*	0.88181818	1.10052140*	0.99753364
2005=>2006	1.02705348*	0.82542250	0.98203342	1.12440193*	1.01240114*	0.93617021	1.26318381*	1.02438093
2006=>2007	0.82607486	0.92786638	1.03935855*	0.89156244	1.01389354*	0.67859867	0.70741015	0.86925208
2007=>2008	0.91502845	0.94622584	0.87968840	0.98591256	0.95904219	1.15794292*	1.00402253*	0.97826613
Average	0.92063591	0.92188779	0.91696224	0.93837395	1.01914705*	0.99317516	1.09127278*	0.97163641

6. CONCLUSIONS AND POLICY IMPLICATIONS

6.1 Conclusions

This paper outlines an original empirical analysis on national defense and economy by incorporating strategy (game-theoretical model) and performance assessment (DEA) with a subjective evaluation of a select group of DMUs' current progress in balancing national defense construction and economic construction. The findings from the analysis lead to the following conclusions.

Inspirations from the equilibrium solutions from Taiwan and South Korea's game

(1) Taiwan: During the 52 years between 1957 and 2008, Taiwan's true national defense expenditure growth rate was greater than that of the equilibrium solution for 23 years (see Table 12 and Figure 3). This suggests that Taiwan's actual national defense expenditure did not exceed the reasonable limit for social economic development.

(2) South Korea: The actual national defense expenditure of South Korea was higher than that of the equilibrium solution for ten years (see Table 12 and Figure. 4). This indicates that South Korea's DMU has been more effective than Taiwan's in terms of national defense resource utilization; it also explains why the nation experienced only three years of negative economic growth (1986, 1997, and 1998) during a period of 48 years.

2. Discussion of the overall productivity of national defense expenditure

(1) With regard to TFP: Only Israel (1.0912) and Japan (1.0191) showed improvement in their average TFP; the remaining five DMUs in our dataset showed decline. In terms of equilibrium solution, the average TFP for South Korea's equilibrium solution was higher than the true value; this is in direct contrast to the situation in Taiwan.

The average TE for all seven DMUs was found to be greater than 1, indicating that all the DMUs in our dataset have adequately utilized their national defense resources. This indicates that the primary cause for decline in TFP; this assumption is consistent with the facts. The average technical change efficiency for all seven DMUs in our dataset were below 1. From this, we can establish that a decline in the overall national defense productivity frontier has been the primary factor behind the decline in TFP.

(2) In terms of TE: India (1.042), Japan (1.032) and Israel (1.000) all showed improvement.

During the analysis, we also established that TE = PTE × SE. In terms of PTE, DMUs that scored above 1 were India (1.037), Japan (1.007), Taiwan (1.005), Taiwan's equilibrium solution (1.000), and Israel (1.000). DMUs whose scale efficiency went through decline included South Korea's equilibrium solution (0.995), Taiwan (0.973), Taiwan's equilibrium solution (0.973), and South Korea (0.995).

This shows that, owing to non-proportional input and output, both Taiwan and Taiwan's equilibrium solution did not reach the optimal scale for return.

If we compare and analyze the relative efficiency of the equilibrium solution to Taiwan's game, Taiwan's actual expenditure, the equilibrium solution to South Korea's game, and South Korea's actual expenditure, it is clear that the TE values for all four of these DMUs were greater than 1 (with the exception of South Korea's equilibrium solution (1.0304), which was greater than South Korea (1.0208); in contrast, Taiwan (1.0151) was greater than Taiwan's equilibrium solution (1.0108). In terms of technical change, both Taiwan and its equilibrium solution were equivalent at (0.9572), whereas South Korea (0.9622) outperformed South Korea's equilibrium solution (0.9606). In terms of TFP, South Korea's equilibrium solution (0.9383) was greater than South Korea (0.9218) and Taiwan (0.9206) was greater than Taiwan's equilibrium solution (0.9169). On the basis of the relative efficiency between the equilibrium solution (derived from the game-theoretic model) and their actual expenditures, the differences between South Korea's and Taiwan's performance reflects the discrepancies between theory and reality; it would be a worthy endeavor to clarify the discrepancies further.

From analyses and comparisons of the national defense expenditure (input) and military capabilities (output) presented in the previous sections, it is clear that the DMUs in our dataset have achieved an outstanding performance in terms of technical efficiency. This indicates that the DMUs in our dataset have all made adequate use of the national defense resources and have outstanding military capabilities.

6.2 Policy Implications

This paper offers significant advantages over other studies that have been conducted in this area by establishing a game-theoretic model between the government and the Ministry of National Defense that moves beyond the limitations of traditional research frameworks. By focusing on the interactive relationship between national defense expenditure, national defense security, and economic growth the analysis can accurately predict a scheme of growth for Taiwan and South Korea that allows the configuration of national defense expenditure and economic construction to reach equilibrium under limited state financial resources. By adopting the DEA method, this analysis is not a purely theoretical discussion (a prevalent trend in previous studies). Instead, the authors attempt to identify the discrepancies between the theoretical values and true values in examining the effectiveness of the approaches adopted by the DMUs in the dataset. Findings from this research will serve as a useful reference for competent authorities in their decision-making processes for effective allotment of state financial resources in their attempt to achieve an effective configuration of equilibrium between national defense and economic construction.

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